DC Circuits

Problems & Solutions

Michael F. McGraw, Ph.D.
DC CIRCUIT ANALYSIS

1. Combine all series and parallel resistors where possible - this reduces the number of loops needed.

2. Pick a direction for each current inside each circuit loop. If the direction is wrong a minus sign in the solution to the equations will point that out.

3. Using Kirchhoff's loop law to write down the voltage drop loop equation for each circuit loop.

   A. If the current goes through a battery in the conventional direction $\rightarrow +$ the voltage is added in with a "+" sign.

   B. If the current goes through a battery in the opposite direction $\rightarrow -$ (charging direction) the voltage is added into the loop equation with a "-" sign.

   C. All resistors are included as "IR" voltage drops with a "-" sign (within their own loop).

4. Some elements in the circuit will have more than one current passing through it. That contribution from other loops needs to be included in the main loop equation.

   \[
   \begin{align*}
   \text{Loop 2 terms: } & -i_2R_2 + i_1R_2 \\
   & -(i_2-i_1)R_2 \\
   \text{Loop 2 terms: } & \quad i_2R_2 - i_1R_2 \\
   & -(i_1+i_2)R_2
   \end{align*}
   \]

   This incorporates the current node equations into the voltage loop equations.
5. There should be as many voltage loop equations as there are current loops

6. Solve these equations for each of the currents

7. Resolve the current in shared segments using conservation of current at the appropriate node

\[ \begin{align*}
    i_1 & \quad i_2 \\
    \downarrow & \quad \downarrow \\
    x & \quad -i_1 - x + i_2 = 0 \\
    & \quad x = i_2 - i_1
\end{align*} \]

- Currents going into a node are negative
- Currents coming out of a node are positive

8. Label the currents in the various segments of the circuit

9. Label the voltage drops across each of the elements of the circuit.

10. Check the numbers. Pick a point of the circuit as ground (i.e., 0 volts). Label the voltages of all the nodes in the circuit relative to ground. Are these consistent?

Do the voltage drops agree with the current and resistance values?
**Example 18.4**

\[ R_1 = 6.0 \, \Omega \]
\[ R_2 = 3.0 \, \Omega \]
\[ R_3 = 10.0 \, \Omega \]
\[ V = 12.6 \, V \]

**Loop 1**

1. \( V - i_1 R_2 - i_1 R_3 + i_2 R_2 = 0 \)
2. \( -i_2 R_2 - i_2 R_3 + i_1 R_2 = 0 \)

**Simplify**

3. \( V - i_1 (R_2 + R_3) + i_2 R_2 = 0 \)
4. \( + i_1 R_2 - i_2 (R_1 + R_2) = 0 \)

**Substitute values**

5. \( 12 - 13i_1 + 3i_2 = 0 \)
6. \( + 3i_1 - 9i_2 = 0 \)

\[ \text{Eqn}(5) \times 3 + \text{Eqn}(6) \]

\[ 36 - 39i_1 + 9i_2 = 0 \]
\[ + 3i_1 - 9i_2 = 0 \]

\[ 36 - 36i_1 = 0 \]
\[ 36 = 36 \, i_1 \]

\[ 1.0 \, A = i_1 \]

Sub into Eqn (6)

\[ + 3(1) - 9i_2 = 0 \]
\[ 3 = 9i_2 \]

\[ \frac{1}{3} A = i_2 \]

**Current per loop**

Path will follow direction of current loop.

\[ \sum i = -i_1 + i_2 + i_3 = 0 \]

\[ i_3 = i_1 - i_2 \]
\[ i_3 = 1.0 - \frac{1}{3} \]
\[ i_3 = \frac{2}{3} A \]

**Voltage drops**

\[ V_3 = 2 \, V \]
\[ V_6 = 1.6 = \frac{8}{5} \, V \]

\[ V_1 = 10 \, V \]

**Power**

\[ P = V I = 12 \, (1) = 12 \, W \]

**Resistors**

\[ P_3 = V_3 i_3 = 2 (\frac{2}{3}) = \frac{4}{3} \, W \]
\[ P_6 = V_6 i_2 = 2 (\frac{1}{3}) = \frac{2}{3} \, W \]
\[ P_1 = V_1 i_1 = 10 \, (1) = 10 \, W \]

\[ 12 \, W \]
EXAMPLE 18-10

Loop 1
\[ 6 - 6i_1 - 12 - 2i_1 + 2i_2 = 0 \]
\[ -6 - 8i_1 + 2i_2 = 0 \]

Loop 2
\[ 12 - 9i_1 - 2i_2 + 2i_1 = 0 \]
\[ 12 - 11i_1 + 2i_1 = 0 \]

\[ -8i_1 + 2i_2 = 6 \]
\[ 2i_1 - 11i_2 = -12 \]

Eqn (1)
Eqn (2)

Simplify

Divide Eqn (1) by 2

\[ -4i_1 + i_2 = 3 \]

Solve for \( i_2 \)

\[ i_2 = 3 + 4i_1 \]  Eqn (3)

Sub into Eqn (2)

\[ 0i_1 - 11i_2 = -12 \]
\[ 2i_1 - 11(3 + 4i_1) = -12 \]
\[ 2i_1 - 33 - 44i_1 = -12 \]
\[ -42i_1 = -12 + 33 = 21 \]
\[ i_1 = - \frac{21}{42} = -0.5 \text{ A} \]

Sub into Eqn (3)

\[ i_2 = 3 + 4i_1 = 3 + 4(-0.5) = 1.0 \text{ A} \]

Node A

\[ i_3 = 1.0 \]
\[ i_1 = 0.5 \text{ A} \]

\[ \sum i = -i_3 + i_1 + i_2 = 0 \]
\[ i_3 = i_1 + i_2 \]
\[ = 0.5 + 1.0 \]
\[ i_3 = 1.5 \text{ A} \]

Voltage Drops

\[ V_6 = i_1 \cdot 6 = (0.5) \cdot 6 = 3 \text{ V} \]
\[ V_2 = i_3 \cdot 2 = (1.5)(2) = 3 \text{ V} \]
\[ V_4 = i_2 \cdot 4 = (1)(4) = 4 \text{ V} \]

Power

Batteries:
\[ P_6 = 6 \cdot i_1 = 6(0.5) = 3 \text{ W} \]
\[ P_{va} = 12 \cdot i_3 = 12(1.5) = 18 \text{ W} \]

Resistors:
\[ P_6 = V_6 \cdot i_1 = 3(0.5) = 1.5 \text{ W} \]
\[ P_2 = V_2 \cdot i_3 = 3(1.5) = 4.5 \text{ W} \]
\[ P_4 = V_4 \cdot i_2 = 4(1) = 9 \text{ W} \]

\[ 15 + 0 \text{ W} \]
DC Circuit Problems

60. Three resistors and two 10.0-V batteries are arranged as shown in the circuit diagram. Which one of the following entries in the table is correct? Power Delivered Power Delivered by Battery 1 by Battery 2

\[
\begin{array}{ccc}
\text{Power Delivered} & \text{Power Delivered} \\
\text{by Battery 1} & \text{by Battery 2} \\
(d) 1.0 \text{ W} & 4.0 \text{ W} \\
\end{array}
\]

\[
\begin{align*}
\text{USE KIRCHHOFF'S LOOP LAW:} \\
(1) \quad & V_1 - 40i_1 - 12i_1 + 12i_2 = 0 \\
(2) \quad & -V_2 - 12i_2 - 10i_2 + 12i_1 = 0
\end{align*}
\]

\[
\begin{align*}
\text{SIMPLIFY:} \\
& -6i_1 + 12i_2 = -10 \\
& +12i_1 - 22i_2 = 10
\end{align*}
\]

\[
\begin{align*}
\text{SIMPLIFY SOME MORE:} \\
(3) \quad & 16i_1 - 6i_2 = 5 \\
(4) \quad & 6i_1 - 11i_2 = 5
\end{align*}
\]

\[
\begin{align*}
\text{SOLVE (4) FOR} \quad i_1 \\
& 6i_1 = 5 + 11i_2 \\
(5) \quad & i_1 = \frac{5}{6} + \frac{11}{6}i_2
\end{align*}
\]

\[
\begin{align*}
\text{SUB (5) INTO (3):} \\
& 16\left(\frac{5}{6}\right) - 6i_2 = 5 \\
& \frac{135}{3} - 6i_2 = 5 \\
& \frac{135 - 5 - 6i}{3} = \frac{-50}{3}
\end{align*}
\]

\[
\begin{align*}
\text{MUL. BY 3:} \\
& 125i_2 = -50 \\
& i_2 = -\frac{50}{125} = -\frac{2}{5} = -0.4 \text{ A}
\end{align*}
\]

\[
\begin{align*}
\text{SUB INTO (5):} \\
& i_1 = \frac{5}{6} + \frac{11}{6}\left(-\frac{2}{5}\right) = \frac{25 - 22}{30} \\
& i_1 = \frac{3}{30} = 0.1 \text{ A}
\end{align*}
\]

\[
\begin{align*}
\text{BATTERIES:} \\
& P_1 = V_1i_1 = 10 \times 0.10 = 1.0 \text{ W} \quad P_2 = V_2i_2 = 10 \times (-0.40) = -4.0 \text{ W}
\end{align*}
\]
61. Three resistors and two batteries are connected as shown in the circuit diagram. Show that the magnitude of the current through the 12-V battery is 0.52 A.

\[ V_1 = 18 \text{V} \quad V_2 = 12 \text{V} \]

**Use Kirchhoff's Loop Laws**

1. \[ V_1 = 25i_1 - V_2 - 16i_1 = 0 \]
2. \[ V_2 - 18i_2 = 0 \]
3. \[ -41i_1 = V_2 - V_1 \]
4. \[ 18i_2 = V_2 \]

From Eqn (3) solve for \( V_1 \)

\[ -41i_1 = 12 - 18 = -6 \]

\[ i_1 = \frac{6}{41} = 146.35 \text{ mA} \]

From Eqn (4) solve for \( V_2 \)

\[ 18i_2 = 12 \]

\[ i_2 = \frac{12}{18} = 0.667 \text{ A} \]

\[ i_2 = 667 \text{ mA} \]

**The current loops are needed in the correct directions.**

**Net current in \( V_2 \)**

\[ i_2 = 667 \text{ mA} \quad V_2 = 12 \text{V} \]

**Net current in \( V_2 \)**

\[ I = 521 \text{ mA} \]

**Node Eqn A**

\[ \Sigma i = i_1 + i_2 + i_3 = 0 \]

\[ i_3 = 2i_2 - i_1 \]

\[ = 667 - 146 \]

\[ i_3 = 521 \text{ mA} \]

**VOLTAGES**

\[ V_A = 12 \text{V} \]

\[ V_{18} = 18(0.667) = 12 \text{V} \]

\[ V_{16} = 16(0.146) = 2.34 \text{V} \]

\[ V_{25} = 25(0.146) = 3.65 \text{V} \]

**Battery Power**

\[ P_1 = V_1i_1 = 18(0.146) = 2.63 \text{W} \]

\[ P_2 = V_2i_3 = 12(0.521) = 6.25 \text{W} \]

\[ \boxed{8.88 \text{ W}} \]

**Power dissipated in resistors**

\[ P_{16} = V_{16}i_1 = 2.34(0.146) = 0.342 \text{W} \]

\[ = i_1^2R = (0.146)^2 \cdot 16 = 0.341 \text{ W} \]

\[ P_{25} = V_{25}i_1 = 3.65(0.146) = 0.533 \text{W} \]

\[ P_{18} = V_{18}i_2 = 12(0.667) = 8.00 \text{W} \]

\[ P_{16} + P_{25} + P_{18} = 0.342 + 0.533 + 8.00 = \boxed{8.88 \text{ W}} \]
The given circuit diagram and equations are as follows:

18 = 12i_1 - 6i_1 + 6i_2 = 0
18 = 18i_1 + 6i_2 = 0
3 = 3i_1 + i_2 = 0
-3i_2 + 2i_1 - 2i_2 - 6i_2 + 6i_1 = 0
21 - 11i_2 + 6i_1 = 0
21 + 6i_1 - 11i_2 = 0

For Node A:

-i_1 - x + i_2 = 0
i_2 - i_1 = x
3A - 2A = 1A = x

For Node B:

i_1 + x - i_2 = 0
x = i_2 - i_1
3A = 2A + x
x = 1A

\[ i_2 = 3i_1 - 3 = 3(2) - 3 = 3A. \]

\[ 21 = -6i_1 + 11(3i_1 - 3) \]

\[ 21 = -6i_1 + 33i_1 - 33 \]

\[ 54 = 27i_1 \]

\[ 2A = i_1 \]
\[ 75V - i_1 - 5i_1 + 5i_2 = 0 \]

Solve Eqn (1) for \( i_1 \):

\[ 6i_1 = 75 + 5i_2 \]

Substitute into Eqn (2):

\[ 24 - 5\left( \frac{75}{6} + \frac{5}{6}i_2 \right) + 17i_2 = 0 \]

\[ 144 - 375 - 25i_2 + 102i_2 = 0 \]

\[ -231 + 77i_2 = 0 \]

\[ i_2 = \frac{231}{77} = +3A \quad \text{Sub. w Eqn (3)} \]

Combine 6, 3, 2:

\[ \frac{1}{R_{eq}} = \frac{1}{6} + \frac{1}{3} + \frac{1}{2} = \frac{1 + 2 + 3}{6} = 1 \]

\[ V_{BC} = 12V \]

**Node A**

\[ i_1 - i_2 - x = 0 \]

\[ x = i_1 - i_2 \]

\[ x = 15 - 3 = 12A \]

\[ i_6 = \frac{12}{6} = 2A \]

\[ i_3 = \frac{12}{3} = 4A \]

\[ i_4 = \frac{12}{5} = 6A \]

\[ i_2 = \frac{12}{12} = 1A \]
EX 18.4

$2.0V = \frac{2.0 - 12}{10}$

$\frac{i}{3} = \frac{-10}{10}$

$i = \frac{10}{10}$

$i = 1.0 \text{A}$

$i_6 = \frac{V_A}{6} = \frac{2}{6} = \frac{1}{3} \text{A}$

$i_5 = \frac{V_A}{3} = \frac{2}{3} = \frac{2}{3} \text{A}$

CURRENT SOLUTIONS VIA NODE EQUATION

$\frac{V_A}{3} + \frac{V_A}{6} + \frac{V_A - 12}{10} = 0$

$V_A \left( \frac{1}{3} + \frac{1}{6} \right) + \frac{V_A - 12}{10} = 0$

$V_A \left( \frac{6 + 3}{18} \right) + \frac{V_A - 12}{10} = 0$

$V_A \left( \frac{9}{18} \right) + \frac{V_A}{10} = \frac{12}{10} = \frac{6}{5}$

$V_A \left( \frac{1}{2} + \frac{1}{10} \right) = \frac{6}{5}$

$\frac{V_A}{6} = \frac{6}{5}$

$V_A = \frac{6}{5} \cdot \frac{10}{6} = 2V$

$V_A = 2.0V$. 

**Example 18-10**

\[ \frac{V_a}{6} + \frac{V_a-12}{2} + \frac{V_a}{9} = 0 \]

Multiply by 18,

\[ 3(V_a-6) + 9(V_a-12) + 2V_a = 0 \]

\[ 3V_a + 9V_a + 2V_a - 18 - 108 = 0 \]

\[ 14V_a = 126 \]

\[ V_a = \frac{126}{14} = \frac{63}{7} = 9.0 \text{V} \]

\[ i_1 = \frac{V_a-6}{6} = \frac{9-6}{6} = \frac{3}{6} = 0.5 \text{A} \]

\[ i_2 = \frac{V_a-12}{2} = \frac{9-12}{2} = -\frac{3}{2} = -1.5 \text{A} \]

\[ i_3 = \frac{V_a}{9} = \frac{9}{9} = 1.0 \text{A} \]

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\[ \frac{V_a-10}{40} + \frac{V_a}{12} + \frac{V_a-10}{10} = 0 \]

Multiply by 120,

\[ 3(V_a-10) + 10V_a + 12(V_a-10) = 0 \]

\[ 3V_a + 10V_a + 12V_a - 30 - 120 = 0 \]

\[ 25V_a = 150 \]

\[ V_a = \frac{150}{25} = 6.0 \text{V} \]

\[ i_1 = \frac{V_a-10}{40} = \frac{6-10}{40} = -\frac{4}{40} = -0.1 \text{A} \]

\[ i_2 = \frac{V_a}{12} = \frac{6}{12} = 0.5 \text{A} \]

\[ i_3 = \frac{V_a-10}{10} = \frac{6-10}{10} = -\frac{4}{10} = -0.4 \text{A} \]
\[ V_{W} = 18 \quad \sum i = i_1 + i_2 + i_3 = 0 \]

\[ i_1 = \frac{V_a - 18}{41} = \frac{18 - 18}{41} = 0 \]

\[ i_1 = -0.146A \]

\[ i_2 = \frac{V_a}{18} = \frac{18}{18} = 1.0A \]

\[ i_3 = \frac{V_a + 21}{5} = \frac{18 + 21}{5} = 3.0A \]

\[ V_{a - 18} + \frac{V_a}{6} + \frac{V_a + 21}{5} = 0 \]

Multiply by 60

\[ 5(V_{a - 18}) + 10V_a + 12(V_a + 21) = 0 \]

\[ 5V_a + 10V_a + 12V_a - 90 + 252 = 0 \]

\[ 27V_a = -162 \]

\[ V_a = -162 \times \frac{54}{9} = -6.0V. \]
\[ \frac{V_a - 75}{1} + \frac{V_a}{5} + \frac{V_a - 24}{12} = 0 \]

Multiply by 60

\[ 60(V_a - 75) + 12V_a + 5(V_a - 24) = 0 \]
\[ 60V_a + 12V_a + 5V_a - 4500 - 120 = 0 \]
\[ 77V_a = 4620 \]
\[ V_a = \frac{4620}{77} = 60.0 \text{ V} \]

\[ i_1 = \frac{V_a - 75}{1} = 60 - 75 = -15 \text{ A} \]

\[ i_2 = \frac{V_a}{5} = \frac{60}{5} = 12.0 \text{ A} \]

\[ i_3 = \frac{V_a - 24}{12} = \frac{60 - 24}{12} = \frac{36}{12} = 3.0 \text{ A} \]