E & M - Basic Physical Concepts

Electric force and electric field

Electric force between 2 point charges:

$|F| = k\,\frac{|q_1|\,|q_2|}{r^2}$

$k = 8.987551787 \times 10^9\ \mathrm{N\,m^2/C^2}$

$\epsilon_0 = \frac{1}{4\,\pi\,k} = 8.854187817 \times 10^{-12}\ \mathrm{C^2/N\,m^2}$

$q_p = -q_e = 1.60217733\,(49) \times 10^{-19}\ \mathrm{C}$

$m_p = 1.672623\,(10) \times 10^{-27}\ \mathrm{kg}$

$m_e = 9.1093897\,(54) \times 10^{-31}\ \mathrm{kg}$

Electric field: $\vec{E} = \frac{\vec{F}}{q}$

Point charge: $|E| = k\,\frac{|Q|}{r^2}$, $\quad \vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots$

Field patterns: point charge, dipole, $\|$ plates, rod, spheres, cylinders,...

Charge distributions:

Linear charge density: $\lambda = \frac{\Delta Q}{\Delta x}$

Area charge density: $\sigma_A = \frac{\Delta Q}{\Delta A}$

Surface charge density: $\sigma_{surf} = \frac{\Delta Q_{surf}}{\Delta A}$

Volume charge density: $\rho = \frac{\Delta Q}{\Delta V}$

Electric flux and Gauss' law

Flux: $\Delta\Phi = E\,\Delta A_\perp = \vec{E}\cdot\hat{n}\Delta A$

Gauss law: Outgoing Flux from S, $\quad \Phi_S = \frac{Q_{enclosed}}{\epsilon_0}$

Steps: to obtain electric field

–Inspect $\vec{E}$ pattern and construct S

–Find $\Phi_s = \oint_{surface} \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$, solve for $\vec{E}$

Spherical: $\Phi_s = 4\,\pi\,r^2\,E$

Cylindrical: $\Phi_s = 2\,\pi\,r\,\ell\,E$

Pill box: $\Phi_s = E\,\Delta A$, 1 side; $\quad = 2\,E\,\Delta A$, 2 sides

Conductor: $\vec{E}_{in} = 0$, $\quad E^{\|}_{surf} = 0$, $\quad E^{\perp}_{surf} = \frac{\sigma_{surf}}{\epsilon_0}$

Potential

Potential energy: $\Delta U = q\,\Delta V \quad 1\ \mathrm{eV} \approx 1.6 \times 10^{-19}\ \mathrm{J}$

Positive charge moves from high V to low V

Point charge: $V = \frac{k\,Q}{r} \quad V = V_1 + V_2 = \ldots$

Energy of a charge-pair: $U = \frac{k\,q_1\,q_2}{r_{12}}$

Potential difference: $|\Delta V| = |E\,\Delta s_{\|}|$,

$\Delta V = -\vec{E}\cdot\Delta\vec{s}$, $\quad V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{s}$

$E = -\frac{dV}{dr}$, $\quad E_x = -\frac{\Delta V}{\Delta x}\Big|_{fix\ y,z} = -\frac{\partial V}{\partial x}$, *etc.*

Capacitances $\quad Q = C\,V$

Series: $V = \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \cdots$, $\quad Q = Q_i$

Parallel: $Q = C_{eq}\,V = C_1\,V + C_2\,V + \cdots$, $\quad V = V_i$

Parallel plate-capacitor: $C = \frac{Q}{V} = \frac{Q}{E\,d} = \frac{\epsilon_0\,A}{d}$

Energy: $U = \int_0^Q V\,dq = \frac{1}{2}\,\frac{Q^2}{C}$, $\quad u = \frac{1}{2}\,\epsilon_0\,E^2$

Dielectrics: $C = \kappa C_0$, $\quad U_\kappa = \frac{1}{2\,\kappa}\,\frac{Q^2}{C_0}$, $\quad u_\kappa = \frac{1}{2}\,\epsilon_0\,\kappa\,E_\kappa^2$

Spherical capacitor: $V = \frac{Q}{4\,\pi\,\epsilon_0\,r_1} - \frac{Q}{4\,\pi\,\epsilon_0\,r_2}$

Potential energy: $U = -\vec{p}\cdot\vec{E}$

Line: $E = \frac{2k\lambda}{R}$

Ring (axis): $E_z = \frac{kQz}{(z^2+a^2)^{3/2}}$

Current and resistance

Current: $I = \frac{dQ}{dt} = n\,q\,v_d\,A$

Ohm's law: $V = I\,R$, $\quad E = \rho J$

$E = \frac{V}{\ell}$, $\quad J = \frac{I}{A}$, $\quad R = \frac{\rho\ell}{A}$

Power: $P = I\,V = \frac{V^2}{R} = I^2\,R$

Thermal coefficient of ρ: $\alpha = \frac{\Delta\rho}{\rho_0\Delta T}$

Motion of free electrons in an ideal conductor:

$a\,\tau = v_d \rightarrow \frac{q\,E}{m}\,\tau = \frac{J}{n\,q} \rightarrow \rho = \frac{m}{n\,q^2\,\tau}$

Direct current circuits $\quad V = I\,R$

Series: $V = I\,R_{eq} = I\,R_1 + I\,R_2 + I\,R_3 + \cdots$, $\quad I = I_i$

Parallel: $I = \frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \cdots$, $\quad V = V_i$

Steps: in application of Kirchhoff's Rules

–*Label currents:* $i_1, i_2, i_3, \ldots$

–*Node equations:* $\sum i_{in} = \sum i_{out}$

–*Loop equations:* "$\sum(\pm\mathcal{E}) + \sum(\mp iR) = 0$"

–*Natural:* "+" for loop-arrow entering – terminal

"–" for loop-arrow-parallel to current flow

RC circuit: if $\frac{dy}{dt} + \frac{1}{RC}\,y = 0$, $\quad y = y_0\exp(-\frac{t}{RC})$

Charging: $\mathcal{E} - V_c - R\,i = 0$, $\frac{1}{c}\,\frac{dq}{dt} + R\,\frac{di}{dt} = \frac{i}{c} + R\,\frac{di}{dt} = 0$

Discharge: $0 = V_c - R\,i = \frac{q}{c} + R\,\frac{dq}{dt}$, $\quad \frac{i}{c} + R\,\frac{di}{dt} = 0$

Magnetic field and magnetic force

$\mu_0 = 4\,\pi \times 10^{-7}\ \mathrm{T\,m/A}$

Wire: $B = \frac{\mu_0\,i}{2\,\pi\,r}$ $\qquad$ **Axis of loop:** $B = \frac{\mu_0\,a^2\,i}{2\,(a^2+x^2)^{3/2}}$

Magnetic force: $\vec{F}_M = i\,\vec{\ell}\times\vec{B} \rightarrow q\,\vec{v}\times\vec{B}$

Loop-magnet ID: $\vec{\tau} = i\,\vec{A}\times\vec{B}$, $\quad \vec{\mu} = i\,A\,\hat{n}$

Circular motion: $F = \frac{m\,v^2}{r} = q\,v\,B$, $\quad T = \frac{1}{f} = \frac{2\,\pi\,r}{v}$

Lorentz force: $\vec{F} = q\,\vec{E} + q\,\vec{v}\times\vec{B}$

Hall effect: $V_H = \frac{F_M\,d}{q}$, $\quad U = -\vec{\mu}\cdot\vec{B}$

Sources of $\vec{B}$ and magnetism of matter

Biot-Savart Law: $\Delta\vec{B} = \frac{\mu_0}{4\pi}\,\frac{i\,\Delta\vec{\ell}\times\hat{r}}{r^2}$, $\quad B = \frac{\mu_0}{4\pi}\,\frac{q\,\vec{v}\times\hat{r}}{r^2}$

$\Delta B = \frac{\mu_0}{4\pi}\,\frac{i\,\Delta y}{r^2}\sin\theta$, $\quad \sin\theta = \frac{a}{r}$, $\quad \Delta y = \frac{r^2\,\Delta\theta}{a}$

Ampere's law: $\mathcal{M} = \oint_L \vec{B}\cdot d\vec{s} = \mu_0\,I_{encircled}$

Steps: to obtain magnetic field

–Inspect $\vec{B}$ pattern and construct loop L

–Find $\mathcal{M}$ and I_{encl}, and solve for $\vec{B}$.

Displ. current: $I_d = \epsilon_0\,\frac{d\,\Phi_E}{dt} = \epsilon_0\,\frac{d\,(E\,A)}{dt} = \frac{d\,Q_A}{dt}$

Magnetism in atom:

Orbital motion: $\mu = i\,A = \frac{e}{2\,m}\,L$

$L = m\,v\,r = n\,\hbar$, $\quad \hbar = \frac{h}{2\,\pi} = 1.06 \times 10^{-34}\ \mathrm{J\,s}$

$\mu_{orbit} = n\,\mu_B$, $\quad \mu_B = \frac{e\,\hbar}{2\,m} = 9.27 \times 10^{-24}\ \mathrm{J/T}$

Spin: $S = \frac{\hbar}{2}$, $\quad \mu_{spin} = \mu_B$

Magnetism in matter:

$B = B_0 + B_M = (1+\chi)\,B_0 = (1+\chi)\,\mu_0\,\frac{B_0}{\mu_0} = \kappa_m\,H$

Ferromagnetic: $\chi \gg 1$ $\qquad$ **Diamagnetic:** $-1 \ll \chi < 0$

Paramagnetic: $0 < \chi \ll 1$, $\quad M = \frac{C}{T}B$

E-FIELD

$E = \sigma/2\epsilon_0$ INSULATOR

$E = \sigma/\epsilon_0$ CONDUCTOR

$E = \sigma/\epsilon_0$ CAPACITOR

V-RING (AXIS): $V = \frac{kQ}{\sqrt{z^2+a^2}}$

V-LINE $\quad V = 2k\lambda\ln\left(\frac{R_{ref}}{R}\right)$

Faraday's law

$$\mathcal{E} = -N\,\frac{d\phi_B}{dt}, \qquad \phi_B = \int \vec{B}\cdot d\vec{A},$$

$$\mathcal{E} = \int \vec{E}\cdot d\vec{s}, \qquad \vec{E} = \frac{\vec{F}_M}{q}$$

Lenz law: Induced $\vec{B}$ opposes change of Φ_B

$$\frac{d\phi_B}{dt} = \frac{d(B\,A_\perp)}{dt} = \frac{dB}{dt}\,A_\perp + B\,\frac{dA_\perp}{dt}$$

Moving rods: $\frac{dA}{dt} = \ell\,v$, $\frac{dA}{dt} = \frac{d}{dt}\left(\frac{1}{2}\,R\cdot R\theta\right)$

Rotating loop: $\frac{dA_\perp}{dt} = \frac{d}{dt}\,(A\,\cos\omega t)$

Cutting B lines $\rightarrow$ change $\phi_B \rightarrow E_{ind} \rightarrow \mathcal{E}_{ind}$

Maxwell equations:

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0}, \qquad \oint \vec{B}\cdot d\vec{A} = 0,$$

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\phi_B}{dt}, \quad \oint \vec{B}\cdot d\vec{s} = \mu_0\,[I + \epsilon_0\,\frac{d\phi_E}{dt}]$$

Inductance

Mutual: $\mathcal{E}_2 = -M_{21}\,\frac{di_1}{dt}$, $M_{21} = M_{12} = \frac{N_2\,\phi_{21}}{i_1}$

Self: $\mathcal{E} = -L\,\frac{di}{dt}$, $L = \frac{N\,\phi}{i}$, $V_L = L\,\frac{di}{dt}$

Long solenoid: $L = \frac{N\,B\,A}{i}$, $B = \mu_0\,n\,i$

Energies: $U_L = \frac{1}{2}\,L\,i^2$, $u_B = \frac{1}{2\,\mu_0}\,B^2$

$$U_C = \frac{1}{2\,C}\,q^2, \quad u_E = \frac{1}{2}\,\epsilon_0\,E^2$$

L C: $V_L + V_C = 0 \Rightarrow L\,\frac{di}{dt} = -\frac{q}{C}$ $q = q_0\cos(\omega\,t + \delta)$,

$$\omega = \sqrt{\frac{1}{LC}}, \quad U_C + U_L = U_{C\,\max} = U_{L\,\max} = U_0$$

Decay Equations: $\frac{dy}{dt} = -a\,y$, $y = y_0\,\exp(-a\,t)$

L R: $\mathcal{E} = V_L + R\,i$, $\frac{dV_L}{dt} + \frac{R\,V_L}{L} = 0$,

$$V_L = \mathcal{E}\,\exp\left(-\frac{Rt}{L}\right), \quad i = \frac{\mathcal{E}}{R}\left[1 - \exp\left(\frac{-R\,t}{L}\right)\right]$$

L R C:

$$Q \approx Q_0\,e^{-\frac{R}{2L}\,t}\cos\omega_d\,t, \quad \omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2\,L}\right)^2}$$

Underdamped, critically damped & overdamped

A C Circuits

Impedance: $[\mathrm{Ohm} \equiv \Omega]$ $\quad Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$

Inductive $X_L = \omega\,L$, Capactive $X_C = \frac{1}{\omega\,C}$

Mean value: $\bar{f}(t) = \frac{1}{T}\int_0^T f(t)\,dt$

$$[\sin\omega\,t]_{rms} = \overline{[\sin^2\omega\,t]}^{\frac{1}{2}} = \overline{[\tfrac{1}{2}\,(1 - \cos 2\,\omega\,t)]}^{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

Electromagnetic waves

Properties of em waves:

$$E = E_m\cos(k\,z - \omega\,t), \quad B = \frac{E}{c}$$

$$v = \frac{dz}{dt} = \frac{\omega}{k} = \lambda\,f = \frac{\lambda}{T}, \quad n = \frac{c}{v}$$

speed of light: $c = \frac{1}{\sqrt{\epsilon_0\,\mu_0}} = 2.99792458 \times 10^8$ m/s

$\vec{B} \perp \vec{E}$**, propagating along:** $\vec{E} \times \vec{B}$

$$u = u_E + u_B, \quad u_E = u_B$$

Poynting vector: $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$, $\bar{S} = \bar{I} = \frac{E_{rms}B_{rms}}{\mu_0}$

Intensity: $I = \frac{P}{A} = \frac{\Delta U}{A\,\Delta z}\,\frac{dz}{dt} = u\,c$

Energy conservation: $\int \vec{S}\cdot d\vec{A} = \frac{dU}{dt} + P_R$

Complete absorption: Momentum $p = \frac{U}{c}$

Pressure: $P = \frac{F}{A} = \frac{\Delta p}{\Delta t}\,\frac{1}{A} = \frac{\Delta U}{c\,\Delta t}\,\frac{1}{A} = u = \frac{S}{c}$

Complete reflection: $P = \frac{2\,U}{c}$, $P = \frac{2\,S}{c}$

Reflection and Refraction

Index of refraction: $\frac{n_1}{n_2} = \frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1}$

Snell's law: $n_1\sin\theta_1 = n_2\sin\theta_2$

Critical angle: $n_2 > n_1$, $n_2\sin\theta_c = n_1\sin 90^\circ$

Total reflection: $\theta > \theta_c$

Mirrors and lenses

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Ray tracing rules:

Mirror: At symm pt S, reflected symmetrically through center of sphere, undeflected. Parallel to axis, converges toward F (or diverges away from F), $f = \frac{R}{2}$.

Lens: Through center of lens, undeflected. Parallel to axis, converges toward F (or diverges away from F)

Image: $q > 0$ (real), $q < 0$ (virtual)

Focal point F**:** at $p = \infty$, $q = f$

$f = \pm|f|$, "+" convergent, "−" divergent

Magnification: $M = \frac{h'}{h} = -\frac{q}{p}$

Refraction at spherical surface: $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$

R is coordinate of center with origin at S, with S the symmetry point of surface on the axis

Lens maker: $\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Two media: $M = \frac{h'}{h} = -\frac{q}{p}\,\frac{n_1}{n_2}$

Huygen's principles:

Points in wave front are sources of next wavelets

Forward tangent surface is next wave front

Interference

Maxima $\phi = 0, 2\,\pi, 4\,\pi, \cdots$; Minima $\phi = \pi, 3\,\pi, 5\,\pi, \cdots$

Double slits: $I_{average} = I_0\cos^2\left(\frac{\phi}{2}\right)$, $\phi = k\,\Delta$.

$\sin\theta = \frac{\Delta}{d}$, $\tan\theta = \frac{y}{L}$, for small θ, $\theta \approx \sin\theta \approx \tan\theta$

Phasor diagram: $\vec{A} = \vec{A}_1 + \vec{A}_2 + \vec{A}_3 + \cdots$

$$A_x = A_{1x} + A_{2x} + A_{3x} + \cdots, \quad A_y = A_{1y} + A_{2y} + \cdots$$

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

First minimum for N **slits:** $\phi = \frac{2\pi}{N}$

Thin film: $\phi = k\,\Delta + |\phi 1_{reflected} - \phi 2_{reflected}|$, $\Delta = 2\,t$

$\phi_{reflected} = \pi$ (denser medium); $=0$ (lighter medium)

Diffraction

Single slit: $I = I_0\left[\frac{\sin\frac{\beta}{2}}{\frac{\beta}{2}}\right]^2$, $\beta = k\Delta$, $\Delta = a\,\sin\theta$

Resolution criterion: $\theta_{criterion} = 1.22\,\frac{\lambda}{D}$

Grating: Principle maxima $\Delta = m\,\lambda$

Polarization

Brewster $(n_1 < n_2)$**:** $n_1\sin\theta_{br} = n_2\sin(\frac{\pi}{2} - \theta_{br})$

Polarizer: $E_{transmit} = E_0\cos\theta$, $I = I_0\cos^2\theta$

Unpolarized light: $\frac{\Delta I}{\Delta\theta} = \frac{I_0}{2\,\pi}$

Transmitted Intensity: $\Delta I' = \Delta I\,\cos^2\theta$

$$I' = \frac{I_0}{2\,\pi}\int_0^{2\,\pi}\cos^2\theta\,d\theta = \frac{I_0}{2}$$

$$P_r = \frac{I}{c} = \frac{E_0 B_0}{2\mu_0 c} = \frac{E_{rms}B_{rms}}{\mu_0 c} = \frac{E_0^2}{2\mu_0 c^2} = \frac{B_0^2}{2\mu_0}\,; \quad \frac{E_0}{B_0} = c$$