

AMATYC
STUDENT MATHEMATICS LEAGUE
TEST #2

INSTRUCTIONS FOR ANSWER SHEET

TO THE STUDENT:

You have one hour to take this test. Scratch paper, which will be supplied, is allowed. Also allowed are calculators, including graphics calculators, provided they do not have a typewriter-like keyboard (such as, for example, the TI-92) or a disk drive. **However, no books, mathematical tables, computers, or questions are permitted during the test.**

You are NOT expected to be able to answer all the questions; do not become discouraged at the difficulty of the questions. Do not waste too much time on one question, but return to it after you finish the others.

Each question has a 2 point value for a correct answer and a penalty of -1/2 point for each wrong answer. There is no deduction for unanswered questions or for wrong answers to questions which are not multiple choice. Do not make any random guesses, but do answer a question if you can eliminate some of the choices.

Put the letter for the best response to each question you answer in the first column on the answer sheet provided.

Are there any questions?

Do not begin until you are told to do so.

1. If $\log_b 12 = 2.5$, which of the following is closest to b ?
A. 2.5 B. 2.7 C. 3.0 D. 3.2 E. 3.4
2. If the ordered pair $(200,3)$ is a solution to the equation $ax + by = c$, which of the following must be a solution to the equation $bx - ay = c$?
A. $(3, -200)$ B. $(-3, 200)$ C. $(3, 200)$ D. $(200, -3)$ E. $(-200, 3)$
3. If $i = \sqrt{-1}$, what is i^{-321} ?
A. 0 B. 1 C. -1 D. i E. $-i$
4. In triangle SML , $m\angle S = 30^\circ$ and $m\angle M = 40^\circ$. If P is chosen on side SM so that segment LP bisects $\angle SLM$, what is $m\angle SPL$?
A. 70° B. 85° C. 90° D. 95° E. 110°
5. Square $MATH$ is placed with vertices A and T on a circle of diameter 20 so that the midpoint of side MH is the circle's center. The area of the square is
A. 40 B. 60 C. 80 D. 90 E. 100
6. Using any number of 7 g and 11 g weights and a balance scale (2 pans joined by an arm), what is the least number of weights needed to balance a 1 g object?
A. 5 B. 6 C. 7 D. 8 E. 9
7. Let $P(x + 2) = 3x + 1$. What value of x satisfies the equation $P(x + 3) = 13$?
A. 3 B. 5 C. 40 D. 16 E. 15
8. How many real solutions does the equation $\sqrt{x^3 - x^2 - x + 2} = x$ have?
A. 1 B. 2 C. 3 D. 4 E. 6
9. The consecutive integers from 5 to 13 are placed in a 3×3 grid, one per square, so that each row, column, and long diagonal sums to the same value. If 6 is in the lower left square and 5 is in the middle right square as shown, what number is in the bottom middle square (the asterisk)?
A. 7 B. 8 C. 9 D. 10 E. 11

		5
6	*	

10. Reverse my age, divide by three, add twenty, and my age you'll see. How old am I? Write your numerical answer in the corresponding space on the answer sheet.
11. Let $a_n = \sum_{k=1}^n k(-1)^k$. What is a_{2003} ?
A. 1001 B. -1001 C. 1002 D. -1002 E. 1003
12. A parabola with vertical axis is shifted 1 unit right and 1 unit up so that it intersects the original parabola at the point $(1,3)$. What is the y -intercept of the original parabola?
A. 2 B. -2 C. 3 D. -3 E. 4
13. A circle of radius 4 lies in the interior of a 60° angle so that it is tangent to both sides of the angle. If a smaller circle is tangent to both sides of the angle and to the larger circle, what is its radius?
A. 1 B. $\frac{3}{2}$ C. 2 D. $\frac{4}{3}$ E. $\frac{5}{3}$

14. If $f(x) = ax + b$ (a and b integers) and $f(f(x)) = 4x + 5$, then $f(2) =$
A. -9 B. -7 C. 0 D. 1 E. 3
15. Let R be the region in the plane consisting of all points (x,y) such that $|x - y| + |x| + |y| \leq 50$. The area of R is
A. 625 B. 1250 C. 1875 D. 2500 E. infinite
16. The side lengths, semiperimeter (half the perimeter), and the area of a triangle form consecutive terms of an arithmetic sequence. What is the perimeter?
A. $5/13$ B. $19/2$ C. 14 D. $49/3$ E. 23
17. A positive integer is called "strange" if its prime factorization consists of exactly two factors, of which exactly one is less than 10. For example, $26 = (2)(13)$ is strange, but $35 = (3)(7)$, $66 = 2(3)(11)$, and $143 = (11)(13)$ are not. Let A , $A+1$, and $A+2$ be the three smallest consecutive positive integers such that all three are strange. What is the larger prime factor of A ?
A. 13 B. 17 C. 19 D. 29 E. 31
18. Three distinct integers whose sum is 42 form a geometric sequence with common ratio an integer. How many such sequences are there?
A. 1 B. 3 C. 5 D. 6 E. 8
19. Suppose x and y are acute angles with $\cos x = \tan y$ and $\cos y = \tan x$. What is the value of $(\sin x)^2 + \sin x$?
A. $\frac{1}{2}$ B. $\frac{-1 + \sqrt{5}}{2}$ C. $\frac{1 + \sqrt{5}}{2}$ D. $\frac{3}{4}$ E. 1
20. A piece of a 10-inch diameter pie is made by cutting 2 inches straight from the edge (not necessarily toward the center), then making a right angle and cutting straight back to the edge. If the second cut is 6 inches long, what is the area in square inches of the piece (to the nearest tenth)?
A. 10.1 B. 10.3 C. 10.5 D. 10.7 E. 10.9

PLEASE PRINT!

AMATYC STUDENT MATHEMATICS LEAGUE

NAME: KEY -- Feb/Mar 2003

COLLEGE: _____

	Student's Responses	Local Corrector
1	B	
2	A	
3	E	
4	D	
5	C	
6	A	
7	A	
8	B	
9	E	
10	48	
11	D	
12	A	
13	D	
14	A	
15	C	
16	C	
17	B	
18	C	
19	E	
20	E	

ROUND: 1

correct = _____

incorrect = _____

blank = _____

= # correct \times 2

- = # incorrect \times $\frac{1}{2}$

= score