MATD 0370 ELEMENTARY ALGEBRA REVIEW FOR TEST 4 (1.1 - 10.1, not including 8.2)

Test 4 also includes review problems from earlier sections so study test reviews 1, 2, and 3 also.

1. Factor completely: $a^2 - b^2$
2. Factor completely: $a^2 + b^2$
3. Factor completely: $a^2 - 2ab + b^2$
4. Factor completely: $a^2 + 2ab + b^2$
5. Factor completely: $a^3 - b^3$
6. Factor completely: $a^3 + b^3$
7. Factor completely: $9ab^3 - 6a^2b^2$
8. Factor completely: $10a^3 + 6a^2b - 2a$
9. Factor completely: $21x^3y^2z^4 - 28x^5y^3$
10. Factor completely: $12x^2 - 3xy - 8x + 2y$
11. Factor completely: $5x^2 + 5xy - x - y$
12. Factor completely: $x^2 - 9x + 20$
13. Factor completely: $x^2 - 15x - 14$
14. Factor completely: $2y^2 + 4y - 48$
15. Factor completely: $2x^2 + 7x + 6$
16. Factor completely: $6y^2 + 7y - 20$
17. Factor completely: $2x^3 - 7x^2 - 30x$
18. Factor completely: $3x^2 - 16x + 5$
19. Factor completely: $9a^2 + 42a + 49$
20. Factor completely: $y^2 - 12y + 36$
21. Factor completely: $25x^2 - 10x + 1$

22. Factor completely: $2x^3 - 18x$

23. Factor completely: $49a^2 - 81b^2$

24. Factor completely: $25x^2 + 36y^2$

25. Factor completely: $64w^3 - 1$

26. Factor completely: $8a^3 + 125c^3$

27. Factor completely: $2x^3 - 54$

NOTE: The equations in Problems 28 – 39 are quadratic equations, which means they can be written in the form $ax^2 + bx + c = 0$ where $a \neq 0$. Remember that you can simplify each side of the equation if possible, then move all terms to one side of the equation so that one side equals zero, and then solve using one of the following two methods: factoring or quadratic formula (when all else fails use this latter method). You will learn about other methods in later math courses. If you have trouble, please ask your instructor.

28. Solve for y: $2y^2 - 8y - 24 = 0$

29. Solve for x: $x^2 + 2x = 15$

30. Solve for a: $a^2 - 4a + 10 = 3a$

31. Solve for m: $1 - 3m^2 = m^2 - 2(m + 1)$

32. Solve for n: $3n^2 + 1 = 9 - 2n$

33. Twice a number is fifteen less than the square of that number. Find the number.

34. The product of two consecutive integers is 210. Find the two integers.

35. The sum of the squares of two consecutive odd integers is three less than eleven times the larger. Find the two integers.

36. A triangle has a base that is 10 cm more than its height. The area of the triangle is 12 square cm. Find the height and the base.

37. Find the dimensions of a rectangular picture whose length is 3 inches shorter than twice its width and whose area is 35 square inches.

38. Find the lengths of the sides of a right triangle if the long leg is 7 cm longer than the short leg and the hypotenuse is 1 cm longer than the long leg.

39. A 113–meter rope is divided into two pieces so that one piece is fifteen meters longer than the other piece. Find the length of each piece.

40. Simplify (reduce), if possible: $\dfrac{x-4}{4-x}$

41. Simplify (reduce), if possible: $\frac{x^2 - x - 6}{3x^2 - 27}$

42. Simplify (reduce your answer if possible): $\frac{x-6}{3} - \frac{4x}{3}$

43. Simplify (reduce your answer if possible): $\frac{6}{y+1} - \frac{y-1}{y+1}$

44. Simplify (reduce your answer if possible): $\frac{6}{y-2} - \frac{5y - y^2}{y-2}$

45. Simplify (reduce your answer if possible): $\frac{2x^2}{3xy^3} \bullet \frac{6y^2}{x^3}$

46. Simplify (reduce your answer if possible): $\frac{2a-4}{a^2 - 6a + 9} \bullet \frac{a^2 - 8a + 15}{4a^2 - 16}$

47. Simplify (reduce your answer if possible): $\frac{3x+9}{x^2 + 5x + 4} \div \frac{x^2 - 2x + 15}{x^2 - 16}$

Review graphing, solving a system of equations by graphing, word problems that can be solved by a system of equations, and basic formulas for area and perimeter. (Pay particular attention to the types of problems I have mentioned that people had problems with on earlier tests in the announcements section of Blackboard.)

ANSWERS:

1. (a – b)(a + b) [NOTE: Know this formula, and use it for factoring a difference of two squares.]

2. Prime (Not Factorable) [NOTE: Know this formula. A sum of two squares is not factorable unless it has a common factor. If it has a common factor, factor it out.]

3. $(a - b)^2$ [NOTE: Know this formula, and use it for factoring a perfect square trinomial with a negative middle term.]

4. $(a + b)^2$ [NOTE: Know this formula, and use it for factoring a perfect square trinomial with a positive middle term.]

5. $(a - b)(a^2 + ab + b^2)$ [NOTE: Know this formula, and use it for factoring a difference of two cubes. If you have factored out common factors first, the trinomial part of the answer will not factor further.]

6. $(a + b)(a^2 – ab + b^2)$ [NOTE: Know this formula, and use it for factoring a sum of two cubes. If you have factored out common factors first, the trinomial part of the answer will not factor further.]

7. $3ab^2(3b – 2a)$

8. $2a(5a^2 + 3ab – 1)$

9. $7x^3y^2(3z^4 – 4x^2y)$

10. $(4x – y)(3x – 2)$

11. $(x + y)(5x – 1)$

12. $(x – 4)(x – 5)$

13. Prime (Not Factorable) [NOTE: $(x – 14)(x – 1) = x^2 – 15x + 14$, *not* $x^2 – 15x – 14$]

14. $2(y + 6)(y – 4)$

15. $(2x + 3)(x + 2)$

16. $(2y + 5)(3y – 4)$

17. $x(2x + 5)(x – 6)$

18. $(3x – 1)(x – 5)$

19. $(3a + 7)^2$

20. $(y – 6)^2$

21. $(5x – 1)^2$

22. $2x(x + 3)(x – 3)$

23. $(7a + 9b)(7a – 9b)$

24. Prime (Not Factorable) - Sum of Squares [NOTE: If you got a different answer, carefully multiply it back together to see that it is not equal to the original problem.]

25. $(4w – 1)(16w^2 + 4w + 1)$

26. $(2a + 5c)(4a^2 – 10ac + 25c^2)$

27. $2(x - 3)(x^2 + 3x + 9)$

28. $y = 6$, or $y = -2$

29. $x = 3$, or $x = -5$

30. $a = 5$, or $a = 2$

31. $m = \frac{1+\sqrt{13}}{4} \approx 1.151$ or $m = \frac{1-\sqrt{13}}{4} \approx -0.651$

32. $n = \frac{4}{3}$ or $n = -2$

33. The number is 5 or –3. (NOTE: You must give both parts of the answer.) To set up this problem, let x = the number. Then the equation is $2x = x^2 - 15$.

34. The integers are 14 and 15, or the integers are –15 and –14. (NOTE: You must give both parts of the answer.) To set up this problem, let x = the first integer. Then x + 1 = the next consecutive integer. The equation is $x(x + 1) = 210$.

35. The integers are 5 and 7. (NOTE: –1.5 and 0.5 are not integers.) To set up this problem, let x = the first integer. Then x + 2 = the next consecutive odd integer. The equation is $x^2 + (x + 2)^2 = 11(x + 2) - 3$.

36. The height of the triangle is 2 cm and the base is 12 cm. To set up this problem, let x = the base. Then x – 10 = the height. The formula for area of a triangle is A = 1/2 bh so the equation is 1/2 x(x – 10) = 12. Hint: Multiply both sides of the equation by 2 to clear the fraction, which makes the equation x(x – 10) = 24.

37. The width of the rectangle is 5 inches and the length is 7 inches. To set up this problem, let x = the width. Then 2x – 3 = the length. The formula for area of a rectangle is A = lw so the equation is (2x – 3)x = 35, or x(2x – 3) = 35.

38. The lengths of the sides of the triangle are 5 cm, 12 cm, and 13 cm. (NOTE: The hypotenuse of a right triangle is always the longest side.) To set up this problem, let x = the length of the short leg. Then x + 7 = the length of the long leg, and x + 7 + 1 = x + 8 = the length of the hypotenuse. Use the Pythagorean Theorem $a^2 + b^2 = c^2$ to write the equation
$x^2 + (x + 7)^2 = (x + 8)^2$.

39. The lengths of the pieces of rope are 49 m and 64 m. To set up this problem, let x = the length of the short piece. Then x + 15 = the length of the long piece. The sum of the two pieces is the total length of rope, which means that

the equation is x + (x + 15) = 113.

40. -1

41. $\dfrac{x+2}{3(x+3)} = \dfrac{x+2}{3x+9}$ (either answer is acceptable)

42. $-x-2$

43. $\dfrac{-y+7}{y+1}$

44. $y-3$

45. $\dfrac{4}{x^2y}$

46. $\dfrac{(a-5)}{2(a+2)(a-3)}$ (you may leave the denominator factored)

47. $\dfrac{3(x-4)}{(x+1)(x-5)}$ (you may leave the numerator and denominator factored)