

RAISING SIGNED NUMBERS TO EXPONENTS

When raising signed numbers to exponents, students are often uncertain about the sign of the result. An important rule to remember is that the exponent applies only to what immediately precedes it. If a number or variable is immediately before the exponent, then only that number or variable is raised to the power. However, if parentheses are immediately before the exponent, then everything inside the parentheses is raised to that exponent.

For example, -3^2 is different from $(-3)^2$. In -3^2 , only the three (not the negative) is squared. That is, -3^2 means the negative of 3^2 . On the other hand, $(-3)^2$ means the square of -3 . In mathematical terms,

$$-3^2 = -3 \cdot 3 = -9 \quad \text{but} \quad (-3)^2 = (-3) \cdot (-3) = 9$$

The same is true with variables:

$$-x^4 = -x \cdot x \cdot x \cdot x = -x^4 \quad \text{but} \quad (-x)^4 = (-x) \cdot (-x) \cdot (-x) \cdot (-x) = x^4$$

Notice that when the exponent is even, the results have different signs, as in the two examples above. However, when the exponent is odd, the results have the same sign.

$$-c^3 = -c \cdot c \cdot c = -c^3 \quad \text{and} \quad (-c)^3 = (-c) \cdot (-c) \cdot (-c) = -c^3$$

In this example, the results are the same because when multiplying an odd number of negatives, the result is negative. Alternatively, when multiplying an even number of negatives, the result is positive. In summary, for any real number n ,

$$(-n)^{\text{odd exponent}} = -n^{\text{that same odd exponent}}$$

$$(-n)^{\text{even exponent}} = n^{\text{that same even exponent}}$$

However, with no parentheses,

$$-n^{\text{any exponent}} = -n^{\text{that same exponent}}$$

What about when evaluating expressions for a given real number? When substituting in a negative value for a variable, the sign is included with the number. Below are three examples of substituting real numbers for a variable.

$$\text{Evaluate } x^4 \text{ for } x = -5 \text{ means } (-5)^4 = (-5) \cdot (-5) \cdot (-5) \cdot (-5) = 5^4 = 625$$

$$\text{Evaluate } -x^4 \text{ for } x = 5 \text{ means } -5^4 = -5 \cdot 5 \cdot 5 \cdot 5 = -625$$

$$\text{Evaluate } -x^4 \text{ for } x = -5 \text{ means } -(-5)^4 = -(-5) \cdot (-5) \cdot (-5) \cdot (-5) = -5^4 = -625$$

In addition to the problems assigned from your Personal Academic Notebook for lesson 1.1, work the following problems:

1. Evaluate each of the following:

a) -7^3

b) $(-3)^4$

c) -6^2

d) $(-2)^5$

2. Simplify each of the following, if possible:

a) $(-x)^6$

b) $(-r)^5$

c) $(-m)^8$

d) $-n^4$

3. Evaluate each expression for the given value:

a) y^4 for $y = -1$

b) c^5 for $c = -3$

c) n^2 for $n = -8$

d) $-x^2$ for $x = 9$

e) $-r^3$ for $r = -5$

f) $-w^2$ for $w = -7$

ANSWERS:

1. a) -343

b) 81

c) -36

d) -32

2. a) x^6

b) $-r^5$

c) m^8

d) $-n^4$

3. a) 1

b) -243

c) 64

d) -81

e) 125

f) -49