MATD 0370 (ELEMENTARY ALGEBRA)
EXERCISE SET 10.1

10.1A SOLVING QUADRATIC EQUATIONS USING THE QUADRATIC FORMULA

Review of Square Roots:
Recall, the **square root** of a number is a number that when multiplied by itself, gives the original number.

For example: $\sqrt{25} = 5$ because $5^2 = 25$

25 is a **perfect square** because it has a **rational** square root. You may use your calculator to evaluate the square root of values that are not perfect squares.
For example, 13 is not a perfect square because it has an **irrational** square root, which means that its square root is a decimal number which never ends or repeats. However, by using a calculator, we can find that $\sqrt{13} \approx 3.606$ if we round to three decimal places.

Quadratic Formula:
In addition to solving by factoring, quadratic equations may be solved by using the Quadratic Formula:

For any quadratic equation written in standard form: $ax^2 + bx + c = 0$ with $a \neq 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Quadratic Formula may be used to solve quadratic equations which may be factored as well as those which cannot be factored or are difficult to factor.

Example 1: Solve using the quadratic formula: $4x^2 + 5x = 6$

First write the equation in standard form: $4x^2 + 5x - 6 = 0$

Then $a = 4$, $b = 5$, and $c = -6$. Substitute these values into the quadratic formula:

$$x = \frac{-5 \pm \sqrt{5^2 - 4(4)(-6)}}{2(4)} = \frac{-5 \pm \sqrt{25 + 96}}{8} = \frac{-5 \pm \sqrt{121}}{8} = \frac{-5 \pm 11}{8}$$

This gives two answers:

$$x = \frac{-5 + 11}{8} = \frac{6}{8} = \frac{3}{4}, \text{ or}$$

$$x = \frac{-5 - 11}{8} = \frac{-16}{8} = -2$$

Note that the equation could have been solved by factoring: $(4x - 3)(x + 2) = 0$

Set each factor equal to zero: $4x - 3 = 0$ or $x + 2 = 0$

Solving each of these equations gives the same solutions as the quadratic formula gave: $x = \frac{3}{4}$ or $x = -2$.

Example 2: Solve using the quadratic formula: $m^2 - m - 2 = 4m - 5$

First write the equation in standard form by subtracting 4m from both sides and adding 5 to both sides of the equation: $m^2 - 5m + 3 = 0$

Then a = 1, b = –5, and c = 3. Substitute these values into the quadratic formula:

$$m = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(3)}}{2(1)} = \frac{5 \pm \sqrt{25 - 12}}{2} = \frac{5 \pm \sqrt{13}}{2}$$

This gives two answers:

$$m = \frac{5 + \sqrt{13}}{2} \approx 4.303 \text{ , or}$$

$$m = \frac{5 - \sqrt{13}}{2} \approx 0.697$$

Note that $\frac{5 + \sqrt{13}}{2}$ and $\frac{5 - \sqrt{13}}{2}$ are exact answers, while 4.303 and 0.697 are approximations to three decimal places.

10.1B MORE APPLICATIONS OF QUADRATIC EQUATIONS

The following examples are applications of quadratic equations.

Example 3: Five times a number is 24 less than the square of that number. Find all such numbers.

Let n = the number.
Then $5n = n^2 - 24$.

Write this equation in standard form by subtracting 5n from both sides:
$0 = n^2 - 5n - 24$, or $n^2 - 5n - 24 = 0$.

You may solve this equation either by factoring or by using the quadratic formula.
Factoring gives $(n + 3)(n - 8) = 0$.

Set each factor equal to zero: $n + 3 = 0$, or $n - 8 = 0$.

Then solve each of these equations to get $n = -3$ or $n = 8$.

The number is –3 or 8 (you must give both answers).

Check the answers in the original problem:
$5(-3) = (-3)^2 - 24$ which gives $-15 = -15$ ✓, and
$5(8) = (8)^2 - 24$ which gives $40 = 40$ ✓.

Example 4: The product of two consecutive odd integers is 195. Find the integers.

Let x = the first odd integer.
Then $x + 2$ = the next odd integer (because odd numbers are two apart).

The equation is $x(x + 2) = 195$.

Distribute the x, and then subtract 195 from both sides of the equation to write the equation in standard form: $x^2 + 2x - 195 = 0$.

Again, you may solve this equation either by factoring or by using the quadratic formula.
Factoring gives $(x + 15)(x - 13) = 0$.
Set each factor equal to zero: $x + 15 = 0$, or $x - 13 = 0$.
Then solve each of these equations to get $x = -15$ or $x = 13$.

For each of these answers, find the next odd integer by adding two to your answer:
For the first answer, –15, the next odd integer is $x + 2 = -15 + 2 = -13$.
For the second answer, 13, the next odd integer is $x + 2 = 13 + 2 = 15$.

The odd integers are –15 and –13, or the odd integers are 13 and 15 (you must give both pairs of answers).

Check the answers by multiplying:
$(-15)(-13) = 195$ ✓, and
$(13)(15) = 195$ ✓.

Example 5: The length of one leg of a right triangle is 9 meters. The length of the hypotenuse is three meters longer than the other leg. Find the length of the hypotenuse and the length of the other leg.

Let x = the length of the other leg in meters.
Then $x + 3$ = the length of the hypotenuse in meters.

Use the Pythagorean Theorem, $a^2 + b^2 = c^2$, to solve the problem:
$x^2 + 9^2 = (x + 3)^2$.
When simplified, the equation becomes $x^2 + 81 = x^2 + 6x + 9$ (don't forget that the square of a binomial is a trinomial).

When x^2 is subtracted from both sides of the equation, the equation becomes:

81 = 6x + 9, which is no longer a quadratic equation because it has no x^2 term.

Solve as a linear equation by subtracting 9 from both sides and then dividing by 6:

72 = 6x, or $\frac{72}{6} = \frac{6x}{6}$, which becomes 12 = x.

Therefore, the length of the other leg is x = 12 meters.
The length of the hypotenuse is x + 3 = 12 + 3 = 15 meters.

Check by using the Pythagorean Theorem:
$12^2 + 9^2 = 15^2$, or
144 + 81 = 225, or
225 = 225 ✓.

In addition to the problems assigned from your Personal Academic Notebook for lesson 10.1, work the following problems.

Solve the quadratic equations below either by factoring or by using the quadratic formula. Give exact answers, and where appropriate, give approximations to three decimal places.

1. $x^2 + 5x = 2$

2. $2y^2 + 4y = 7y + 8$

3. $3m^2 + 1 = 9 - 2m$

4. $5 + 9x = 2x^2$

5. $n^2 - 7n + 5 = 8$

6. $10x^2 - 15x = 0$

7. $5y^2 - 10y + 1 = y + 4$

8. Four times a number is 12 less than the square of that number. Find all such numbers.

9. The product of two consecutive odd integers is 143. Find the two integers.

10. The sum of the squares of two consecutive integers is nine less than ten times the larger. Find the two integers.

11. The length of a rectangular garden is 3 feet longer than the width. If the area of the garden is 88 square feet, find the dimensions of the garden.

12. A triangle has a base that is 2 cm longer than its height. The area of the triangle is 12 square cm. Find the lengths of the height and the base of the triangle.

13. For an experiment, a ball is projected with an initial velocity of 48 feet/sec. Neglecting air resistance, its height H, in feet, after t seconds is given by the formula
$H = 48t - 16t^2$
How long will it take for the ball to hit the ground? (Hint: $H = 0$ when it hits the ground.)

14. The length of one leg of a right triangle is 8 inches. The length of the hypotenuse is four inches longer than the other leg. Find the length of the hypotenuse and the length of the other leg.

15. A water pipe runs diagonally under a rectangular garden that is one meter longer than it is wide. If the pipe is 5 meters long, find the dimensions of the garden.

ANSWERS:

1. $x = \dfrac{-5+\sqrt{33}}{2} \approx 0.372$, or $x = \dfrac{-5-\sqrt{33}}{2} \approx -5.372$

2. $y = \dfrac{3+\sqrt{73}}{4} \approx 2.886$, or $y = \dfrac{3-\sqrt{73}}{4} \approx -1.386$

3. $m = \dfrac{4}{3}$, or $m = -2$

4. $x = -\dfrac{1}{2}$, or $x = 5$

5. $n = \dfrac{7+\sqrt{61}}{2} \approx 7.405$, or $n = \dfrac{7-\sqrt{61}}{2} \approx -0.405$

6. $x = 0$, or $x = \dfrac{3}{2}$

7. $y = \dfrac{11+\sqrt{181}}{10} \approx 2.445$, or $y = \dfrac{11-\sqrt{181}}{10} \approx -0.245$

8. The number is –2 or 6. [Equation is $4x = x^2 - 12$]

9. The odd integers are 11 and 13, or the odd integers are –13 and –11.
[Equation is $x(x + 2) = 143$]

10. The consecutive integers are 0 and 1, or the consecutive integers are 4 and 5.
[Equation is $x^2 + (x + 1)^2 = 10(x + 1) - 9$]

11. The width is 8 feet, and the length is 11 feet. Note that dimensions of geometric figures cannot be negative. [Equation is $x(x + 3) = 88$]

12. The height is 4 cm, and the base is 6 cm. [Equation is $\dfrac{1}{2}x(x + 2) = 12$]

13. The ball will hit the ground in 3 seconds. [Equation is $0 = 48t - 16t^2$]

14. The other leg is 6 inches, and the hypotenuse is 10 inches.
[Equation is $x^2 + 8^2 = (x + 4)^2$]

15. The width is 3 meters, and the length is 4 meters.
[Equation is $x^2 + (x + 1)^2 = 5^2$]