

**Motion Problems**

Problems in this exercise can be solved using systems of equations with two variables like problems in the software and book in 5.2 or using one equation with one variable as the solutions show. The answers will be the same using either method.

**1. The table**

- Columns: Rate, Time, and Distance
- List on left side is all moving bodies

	Rate	Time	Distance
(moving body 1)			
(moving body 2)			

**2. Information**

- Fill in given values that perfectly match an entry in the table
- Identify the quantity that the problem is asking for. Is it a rate, a time, or a distance? This will determine what column to place your variable in. You may want to use a related quantity (i.e.  $x$  and  $x - 2$ ) to fill in multiple entries in the same column. Use clues from the problem to figure out a related quantity that makes sense.
- At this point, two columns should be filled in completely. Fill in the remaining column using the relation  $\text{distance} = \text{rate} \times \text{time}$

**3. The equation**

- An equation comes from entries in *one column* of the table, using information stated in the problem.
- Typical kinds of equations:
  - Distances set equal to each other. Ask yourself whether it is logical that the distances should be the same. If so, you probably want to use  $\text{dist}_1 = \text{dist}_2$
  - Distances are added together and set equal to a given value (total distance). Are the two bodies moving toward each other from a given distance apart? Are they moving away from each other until they reach a given distance? If so, you probably want to use  $\text{dist}_1 + \text{dist}_2 = \text{total dist}$

- iii) One distance is a specified amount greater than the other. If this is the case, you probably want to use  $dist_1 = dist_2 + A$

**I. Motion problems in which you calculate rates**

A. Jane walks 0.5 miles per hour faster than her friend Alice. If Jane covers the same distance in 0.5 hours that Alice covers in 0.6 hours, how fast does each of them walk?

**1. The table**

	Rate	Time	Distance
Jane			
Alice			

**2. Information**

- a. Values that perfectly match an entry in the table
  - i) *Jane walks 0.5 hours*
  - ii) *Alice walks 0.6 hours*
- b. The quantity that the problem is asking for:
  - i) *"How fast does each of them walk?" This is asking for a rate.*
  - ii) *"Jane walks 0.5 miles per hour faster than her friend Alice." Use this sentence to define a related quantity in the Rate column*
- c. Fill in the remaining column using the relation  $\text{distance} = \text{rate} \times \text{time}$

**3. The equations**

- a. *"Jane covers the same distance ... that Alice covers" This is saying that the distances are equal. Make an equation out of the entries in the Distance column.*

B. John's bicycle speed is 6 mph faster than Nathan's. In 3 hours John covers 32 miles more than Nathan covers in 2 hours. What are the bicycle speeds of both John and Nathan?


**II. Motion problems in which you calculate time**

- C. A train leaves San Francisco, traveling south toward Los Angeles at a rate of 60 miles per hour. Half an hour later a train leaves Los Angeles, traveling north toward San Francisco at a rate of 85 miles per hour. The distance from San Francisco to Los Angeles is 400 miles. How long after the Los Angeles train leaves will they pass each other?

	Rate	Time	Distance
SF train			
LA train			

- D. A boy throws a ball to a dog 30 feet away, at a ground speed of 16 feet per second. The dog immediately runs toward the ball at a rate of 9 feet per second. How much time passes before the dog catches the ball?


### Mixture problems

#### 1. The table

a. Column labels depend on the problem. Below are some examples.

	Column 1	Column 2	Column 3
components of the mixture	strength	volume (gallons, oz., etc.)	total content
	price per pound, oz., etc.	weight (pounds, oz., etc.)	total price
	price per unit	units	total price
	A per B	B	A
Mixture	Column 1 $\times$ Column 2 = Column 3		

#### 2. Information

a. Fill in given values that perfectly match an entry in the table

a. Identify the quantity that the problem is asking for.

- i) Is it a strength? a volume? a weight? a price per unit? a number of units? etc. This will determine what column to place your variable in.
- ii) You may want to use a related quantity (i.e.  $x$  and  $12 - x$ ) to fill in multiple entries in the same column.
- iii) Make sure your entries in Column 2 satisfy the relation Row 1 + Row 2 = Row 3. This can help determine an appropriate related quantity to use.

b. At this point, columns 1 and 2 should be filled in completely. Fill in Column 3 using the relation Column 3 = Column 1  $\times$  Column 2. It is important that your columns be appropriately labeled for this step to be valid. Examples:

$$\begin{aligned} \text{strength} \times \text{volume} &= \text{total content}, \\ \text{price per pound} \times \text{weight (pounds)} &= \text{total price} \end{aligned}$$

#### 3. The equation

Create an equation from the entries in column 3. Use the relation: Row 1 + Row 2 = Row 3.

*note: this relation holds for columns 2 and 3, but not for column 1*

**Mixture problems in which you calculate a volume**

- E. You want to make orange-flavored black tea by mixing black tea with orange-spice herbal tea. How much orange-spice herbal tea at \$14.25 per pound should you mix with 2 pounds of black tea at \$18.95 per pound to obtain a mixture whose total price is \$48.00?

	Price per pound	Pounds of tea	Total price of tea
black tea			
orange-spice herbal tea			
mixture			

*Note: the lower-left box is not used here because a price-per-pound of the mixture is not supplied, and is not necessary to calculate. Only use this box if the value for it is explicitly stated, or if it is the quantity you are asked to find. In all other cases it can be ignored.*

- F. You are at a friend's house and see her open a brand new 32 oz carton of milk with 1% milkfat, mix some whole milk (with 4% milkfat) with it and then pour you a glass. You taste it and, being an expert on milk, detect that it contains 1.5% milkfat. How much whole milk did your friend pour in?


**ANSWERS:**

**I. Motion problems in which you calculate rates**

- A. Jane walks 0.5 miles per hour faster than her friend Alice. If Jane covers the same distance in 0.5 hours that Alice covers in 0.6 hours, how fast does each of them walk?

	Rate	Time	Distance
Jane	$x+0.5$	0.5	$0.5(x+0.5)$
Alice	$x$	0.6	$0.6x$

$$\text{equation: } 0.5(x + 0.5) = 0.6x$$

*solution: Jane walks 3 mph and Alice walks 2.5 mph*

- B. John's bicycle speed is 6 mph faster than Nathan's. In 3 hours John covers 32 miles more than Nathan covers in 2 hours. What are the bicycle speeds of both of them?

	Rate	Time	Distance
John	$x+6$	3	$3(x+6)$
Nathan	$x$	2	$2x$

$$\text{equation: } 3(x + 6) = 2x + 32$$

*solution: John's speed is 20 mph and Nathan's speed is 14 mph.*

**II. Motion problems in which you calculate time**

- C. A train leaves San Francisco, traveling south toward Los Angeles at a rate of 60 miles per hour. Half an hour later a train leaves Los Angeles, traveling north toward San Francisco at a rate of 85 miles per hour. The distance from San Francisco to Los Angeles is 400 miles. How long after the Los Angeles train leaves will they pass each other?

	Rate	Time	Distance
SF train	60	$x$	$60x$
LA train	85	$x-0.5$	$85(x-0.5)$

$$\text{equation: } 60x + 85(x - 0.5) = 400$$

*solution: They will pass 2.55 hours after the Los Angeles train leaves.*

ANSWERS:

**II. Motion problems in which you calculate time**

- D. A boy throws a ball to a dog 30 feet away, at a ground speed of 16 feet per second. The dog immediately runs toward the ball at a rate of 9 feet per second. How much time passes before the dog catches the ball?

	Rate	Time	Distance
ball	16	x	16x
dog	9	x	9x

*equation:*  $16x + 9x = 30$

*solution:* The dog catches the ball 1.2 seconds later.

**Mixture problems where you calculate one volume**

- E. You want to make orange-flavored black tea by mixing black tea with orange-spice herbal tea. How much orange-spice herbal tea at \$14.25 per pound should you mix with 2 pounds of black tea at \$18.95 per pound to obtain a mixture whose total price is \$48.00?

	Price per pound	Pounds of tea	Total price of tea
black tea	18.95	2	18.95*2
orange-spice herbal tea	14.25	x	14.25x
mixture		2+x	48.00

*equation:*  $18.95(2) + 14.25x = 48$

*solution:* You should use 0.71 pounds of orange-spice herbal tea in the mix.

- F. You are at a friend's house and see her open a brand new 32 oz carton of milk with 1% milkfat, mix some whole milk (with 4% milkfat) with it and then pour you a glass. You taste it and, being an expert on milk, detect that it contains 1.5% milkfat. How much whole milk did your friend pour in?

	Milkfat %	Volume (oz.)	Total milkfat
1% milkfat milk	.01	32	32*.01
whole milk	.04	x	.04x
mixture	.015	32+x	.015(32+x)

*equation:*  $32(0.01) + 0.04x = 0.015(32 + x)$

*solution:* Your friend poured in 6.4 oz. of whole milk.