Solutions to Application Problems assigned in class (set 1) - Revised (I think they are correct now...)

1) Find the area between $\sin x$ and $\cos x$ (one "cell"):

$$
\int_{-\pi/4}^{\pi/4} (\cos x - \sin x) \, dx
$$

$$
2 \sqrt{2}
$$

2) Find the volume of a pyramid with an equilateral triangle base of side length 2 and height 6:

$$
\int_0^6 \frac{\sqrt{3}}{4} \left( \frac{y}{3} \right)^2 \, dy
$$

$$
2 \sqrt{3}
$$

3) Find the volume of the solids of revolution:

a. between $y = x^2$ and $y = x^4$, about $x = 7$:

$$
2 \pi \int_{-1}^{1} (7 - x) \left( x^2 - x^4 \right) \, dx
$$

$$
\frac{56 \pi}{15}
$$

$$
\pi \int_0^1 \left( (7 - \sqrt{y})^2 - (7 - \sqrt{y})^2 \right) \, dy + \pi \int_0^1 \left( (7 + \sqrt{y})^2 - (7 + \sqrt{y})^2 \right) \, dy
$$

$$
\frac{56 \pi}{15}
$$

b. between $y = x^3$ and $y = 4x$, about about $y = b$ (I said $y = 20$ in the morning class and changed it to something else, which I don't remember, in the afternoon class; this formula is valid for any $b \geq 8$.

$$
\pi \left( \int_{-2}^{0} \left( b - 4x \right)^2 - \left( b - x^3 \right)^2 \right) \, dx + \int_0^2 \left( \left( b - x^3 \right)^2 - \left( b - 4x \right)^2 \right) \, dx
$$

$$
16b \pi
$$

$$
2 \pi \left( \int_{0}^{8} \left( \frac{y}{4} - \sqrt{y} \right) \, (b - y) \, dy + \int_0^8 \left( \sqrt{y} - \frac{y}{4} \right) \, (b - y) \, dy \right)
$$
c. between \( y = \ln x, y = 0, x = 2 \), about \( x = -3 \): (note: Log here means natural log here; you may approximate the integral numerically)

\[
2 \pi \int_{1}^{2} (x + 3) \left( \log[x] \right) \, dx
\]

11.2794

\[
\pi \int_{0}^{\log[2]} \left( 25 - (e^y + 3)^2 \right) \, dy
\]

11.2794

4) Find work:

a. Roll up a 40 ft chain 1/4 of the way, with a 20 lb bucket on the end (chain weighs 2 lb/ft)

\[
\int_{30}^{40} 2 \left( 40 - y \right) \, dy + 10 \left( 30 \right) \left( 2 \right) + 10 \left( 20 \right)
\]

900

b. Pump all the water out of a cone (with sharp end down) with radius 2 ft, height 8 ft

\[
62.4 \pi \int_{0}^{8} \left( \frac{y}{4} \right)^2 \left( 8 - y \right) \, dy
\]

4182.09

c. Pump half the water out of a sphere of radius 5 ft (assume sphere is full)

\[
62.4 \pi \int_{0}^{5} \left( \sqrt{25 - y^2} \right)^2 \left( 5 - y \right) \, dy
\]

51050.9

5) Find the fluid force ("hydrostatic force"): 
a. $9.8 \cdot 1000 \int_0^{\sqrt{3}} (3 - y) \left( \frac{2y}{\sqrt{3}} \right) \, dy = 31322.3$

b. $62.4 \int_{-3}^{3} (8 - y) \sqrt{9 - y^2} \, dy = 7057.27$

c.
\[ 62.4 \int_{0}^{2} (4 - y) \left(2 \left(\frac{y}{2}\right) + 4\right) \, dy \]

1830.4