

Comparing Measurements. Orders of Magnitude

I've had many students who tell me that they're happy in math classes as long as someone tells them exactly what the procedure is, gives some examples, and then the homework and test problems are just like those examples. Those students typically don't like word problems much, because each one seems so different and it's hard to pick out the relevant information.

So, the bad news here is that using mathematics in the real world – and in the biotechnology real world – is mostly word problems. The good news is that we'll spend quite a lot of time looking at those word problems and helping you find the critical ideas in each one, so that you can see the patterns and become confident in using math in biotechnology situations.

We don't have much time in this short course to do this, so we need you to do some work on your own. In particular, you'll need to review some standard math techniques, like laws of exponents, scientific notation, measurement conversions, solving linear equations, and solving proportion problems. Our book has excellent overall summaries of these, with problems and solutions. I will give you guidance on what to review and how to review it. Our class discussions will focus on a few mathematical topics that you may not have seen before and, mostly, on how to use these techniques to solve a variety of biotechnology problems.

Here's an idea to think about to start. **How big is a typical virus? Don't just quote a number, but be able to talk about how big that is in comparison to other things you're familiar with.**

So what's some math to help us? Answer: The idea of orders of magnitude. Consider this example: Here's a meter stick (like a yard stick, but a meter instead.) I'll use it to measure myself. Notice that I'm about 1.5 meters tall. (We could measure more accurately, but for this purpose, we don't need to.) Now let's think of how we'd measure the length of an ant. Remembering how small they are, and looking at these measures, I think that it might be about 2 millimeters long.

To compare these numbers, we could either subtract them or divide them to get a comparison. Of course, to get a meaningful answer from either subtraction or division, we have to write them in the same units.

Subtracting these two numbers isn't very satisfactory, since the difference is about 1.5 meters. We'd have to measure my height a lot more precisely to get anything interesting here. That isn't surprising – it is usually the case when comparing measured numbers of things which are very different in size.

So we will compare them by dividing. Now, of course, the measurements we did here aren't very precise, so that could cause a big difference in the resulting ratio – particularly if the one in the denominator isn't very precise. So we often don't want to give the ratio very precisely. People handle that by just rounding it off to the nearest power of ten. We call that the order of magnitude of the number. When thinking of the order of magnitude, we divide, and then round it off to the nearest power of ten.

So here, $1.5 \text{ meters} / 2 \text{ mm} = 1500 \text{ mm} / 2 \text{ mm} = 750$. When we round to the nearest power of ten, that's $10^3 = 1000$. So the order of magnitude of the ratio is 3. Or, another way of wording it is that the height of a person is 3 orders of magnitude more than the length of an ant.

It is also fairly common to just round off each of the measurements to a power of ten in the first place (before dividing) and then divide those, which, because of the laws of exponents, is the same as subtracting the exponents.

If we wanted to discuss the comparison of weights, the answer wouldn't necessarily be the same as this. Can you see why that might be true?

So here's a fun question for you to answer for tomorrow. **How big is a virus? How many orders of magnitude different from an ant? From a person? Here think of "length," not weight or volume or another measure.**

A hint: Search the Internet for "Orders of magnitude" and see if you can quickly find something that tells you the size of a virus to use here. (When I did a Google search I found several sites with this information.) Spend no more than 5 minutes on this. **Second question: "Why is it reasonable to believe that the site you found is giving accurate information?"**