Eq 4. Objectives:

1. Simplify algebraic expressions with one variable using the distributive law. Examples: "Simplify 3(x + 7)" and "Simplify 4(x – 9)".
2. Solve equations for which you must use the distributive law to simplify one side and check the solutions. Examples: "Solve 4(x – 5) = 60" and "Solve 6(3x + 7) = 132" and "Solve –2(3x – 5) = 70" and "Solve 60 = –4(2x + 7)".
   Comments: Use integers for all coefficients and solutions. No fractions or decimals.

Eq. 4. Explanation and Examples

Read the following discussion about language. This is very important!

Language: In algebra, it is important to distinguish between expressions and equations.
Equations: In this series of supplements, we have been solving equations, which are algebraic "sentences." They are true for some values of the variable. The question is to find out for which values of the variable they are true. We say "solve." Examples: "Solve 2x = 18" and "Solve 3x – 7 = 14".
Expressions: Algebraic expressions do NOT have an equal sign in them. We are asked to "simplify" expressions. We cannot solve them in the same sense as we solve equations. Examples: "Simplify 2x + 3x" or "Simplify 7 * 6 – 8".

We have already done quite a lot of work in this course on simplifying expressions. But there hasn't been much work on using the associative, commutative, and distributive laws on expressions that include variables.

Lots of examples of simplifying expressions with variables are included here.

First, read Examples 1 and 2 and the paragraph right after Example 2. Pay particular attention to which operations are multiplication and which are addition.

Simplifying certain algebraic expressions: The important points to be made in this part of the lesson are when and how to use the distributive law.

Example 1: Simplify 4 (6x)

Solution: 4 (6x)
= 4 * (6 * x) Clarify what each operation is.
= (4 * 6) * x Use the associative law of multiplication, since the operation inside the parentheses was multiplication.
= 24 * x
= 24x

Example 2: Simplify 8 (x + 6)

Solution: 8 (x + 6)

= 8 * (x + 6)	Clarify what each operation is.
= 8 * x + 8 * 6	Use the distributive law since the operation inside the parentheses is addition.
= 8x + 48	Simplify each term

When to use the distributive law: Look carefully at Examples 1 and 2. Both have parentheses. The crucial difference is that the operation inside the parentheses in Example 1 is multiplication. The **distributive law does not apply in Example 1**, because the distributive law is used to multiply the sum or difference of two numbers by a third number. In Example 1, there is no sum or difference. The associative law is appropriate for Example 1 because it tells us about the result of multiplying three quantities.

For Examples 3 and 4, write each example and try to do it yourself before reading the solution.

It is not crucial that you show all the same steps as given here. By the end of this lesson, you'll do problems like these with just one step.

(You have to THINK the steps in between to know whether to use the associative law or distributive law!)

Example 3: Simplify 7(4x – 8)

Solution: 7(4x – 8)

= 7 * (4x – 8)	Clarify what each operation is.
= 7 * 4x – 7 * 8	Use the distributive law, since the operation inside the parentheses is subtraction.
=7 * (4 * x) – 56	Clarify what each operation is.
= (7 * 4) * x – 56	Use the associative law of multiplication.
= 28x – 56	Simplify.

Example 4: Simplify 5(-6x)

Solution: 5(-6x)

= 5 * (–6 * x)	Clarify what each operation is.
= (5 * (–6)) * x	Use the associative law of multiplication, since the operation inside the original parentheses was multiplication.
= –30 x	Simplify.

Is it clear to you how to tell whether to use the associative law and when to use the distributive law?

If not, please ask your instructor now. This is very important!

For Examples 5, 6, and 7, try to do the problem first before you look at the solution.

Example 5: Simplify 8(-3x + 5)

Solution: 8(-3x + 5)

= 8 * (-3x + 5)	Clarify what each operation is.
= 8 * (-3 * x) + 8 * 5	Use the distributive law.
= (8 * (-3)) * x + 40	Use the associative law of multiplication and simplify.
= -24x + 40	Simplify.

Example 6: Simplify 12 (6 – 7x)

Solution: 12 (6 – 7x)

= 12 * (6 – 7x)	Clarify what each operation is.
= 12 * 6 – 12* 7x	Use the distributive law. (What operation was in the parentheses?)
= 72 – 84 x	We used the associative law for multiplication to find that 12*7x = (12*7)*x = 84x

What steps must be shown? Some of the steps listed above were included just to make very clear which operations are being used at each step so that we can see whether to use the associative or distributive law in each step. After this becomes clear to you, you may omit some of the steps that were shown in the previous examples. The following examples illustrate this.

Example 7: Simplify 12(3x – 8)

Solution: 12(3x – 8)

= 12 * 3x – 12 * 8	Use the distributive law.
= 36x – 96	Use the associative law to find 12*3x.

Examples 7 – 10 illustrate the correct use and notation for multiplication by a negative number in front.

Read all three of these examples and pay careful attention to the notation. Using correct and careful notation on problems like these is very important.

What about negative numbers? The following examples illustrate using the distributive law with negative numbers in various places. Notice that in some cases there are two equally correct ways to write the final answer.

Example 8: Simplify –2(7x + 5)

Solution: –2(7x + 5)
= –2 * 7x + (–2) * 5 Use distributive law. (What operation was inside the parentheses?)
= –14x + (–10) Notice the signs. This is a correct final answer.
–14x – 10 is also a correct final answer.
Both possible final answers are equally correct.

Example 9: Simplify –4(7 – 5x)

Solution: –4 (7 – 5x)
= –4 * 7 – (–4)*5x Use the distributive law. (What operation was in the parentheses?)
= – 28 – (–20x) Notice the –4*5x is –20x. And that is subtracted.
= – 28 + (+20x) To subtract a negative number, we add the opposite.
= – 28 + 20x

Example 10: Simplify –5 (–3x + 7)

Solution: = –5 (–3x + 7)
= –5 * (–3x) + (–5)*7 Use the distributive law.
= + 15x + (–35)
= 15x – 35 It is also correct to give the final answer as 15x + (–35)

Think back to the discussion at the very beginning of this section on expressions versus equations. Reread it now if you have forgotten it.

Now we will use the distributive law to simplify expressions IN equations. Read through Examples 11 and 12.

Solving equations: Some equations include expressions that we must simplify using the distributive law. We simplify these FIRST before we start doing the same thing to both sides of the equation.

Example 11: Solve 3(x – 7) = 9

Solution: 3(x – 7) = 9
3 * x – 3 * 7 = 9 Use the distributive law on the left-hand side.
3x – 21 = 9 Simplify.
3x – 21 + 21 = 9 + 21 Add the same number to both sides.
3x + 0 = 30

$3x = 30$
$(1/3) * 3x = (1/3) * 30$ Multiply both sides by the same nonzero number.
$(3/3) * x = 30/3$
$1 * x = 10$
$x = 10$

Example 12: Check the solution to the previous example.

Check: $3(x - 7) = 9$
$3(10 - 7) \;?\; 9$ Replace the variable with the solution and the = with ?
$3(3) \;?\; 9$ Simplify
$9 \;?\; 9$ Simplify
$9 = 9$ Notice that they are equal so replace the ? with =
It checks!

Copy the problem in each of Examples 13 - 16 and try to do it without looking at the solution. Then check your work by looking at the solution.

Example 13: Solve $95 = 5(x - 4)$

Solution: $95 = 5(x - 4)$
$95 = 5 * x - 5 * 4$ Use the distributive law on the right-hand side.
$95 = 5x - 20$
$95 + 20 = 5x - 20 + 20$ Since we want to isolate the term with the variable, we need to "get rid of" the 20 that is subtracted from 5x. We do that by adding 20 to both sides.
$115 = 5x - 0$ Notice that we could write either – 0 or + 0 here.
$115 = 5x$
$(1/5) * 115 = (1/5) * 5x$ We multiply by the reciprocal of the 5 so we'll get just x.
$23 = (5/5) * x$
$23 = x$ **It is fine to leave the solution like this. It is also correct to write another step and say $x = 23$.**

Example 14: Solve $-2(4x - 9) = 98$

Solution: $-2(4x - 9) = 98$

$-2(4x + (-9)) = 98$

$-2*4x + (-2)(-9) = 98$ Use the distributive law.

$-8x + 18 = 98$ Simplify

$-8x + 18 - 18 = 98 - 18$ Show what you subtract from both sides.

$-8x = 80$

$(-1/8) * (-8x) = (-1/8) * 80$ Show what you multiply both sides by.

$x = -80 / 8$

$x = -10$

What steps must we write? By this point in the course, it is possible that you will be able to do *some* of the steps in these solutions in your head correctly and do not need to write down every step. If you find this at all confusing, or if you begin to make mistakes, go back to writing down all the steps.

The next examples illustrate the minimal work you must show in this course when solving an equation.

Example 15: Solve $18 - 3(4x - 9) = 63$

Solution: $18 - 3(4x - 9) = 63$

$18 - 12x + 27 = 63$ Use the distributive law and simplify.

$45 - 12x = 63$

$-12x + 45 - 45 = 63 - 45$ Show what you subtract from both sides.

$-12x = 18$

$(-1/12) * (-12x) = (-1/12) * 18$ Show what you multiply both sides by.

$x = -18 / 12$

$x = -3/2$

Example 16: Solve $15x - 4(x - 9) = 82$

Solution: $15x - 4(x - 9) = 82$

$15x - 4x + 36 = 82$ Use the distributive law and simplify.

$11x + 36 - 36 = 82 - 36$ Show what you subtract from both sides.

$11x = 46$

$(1/11) * (11x) = (1/11) * 46$ Show what you multiply both sides by.

$x = 46 /11$

$x = 4\ 2/11$ The mixed number equivalent to the improper fraction.

Read through Example 17 and the solution.

The following example shows the minimal steps you must show when solving a word problem in this course. In later courses, you may be able to write less for solutions to word problems, but in this course it is particularly important to show how you develop the equation to solve.

Example 17: If I take a particular number, add 7 to it, and then take twice the result, I get 22. What is the number?

Solution: Let n be the number.
Then n + 7 is the result when I add 7 to it.
Then twice the result is 2(n + 7).
So the problem says 2(n + 7) = 22
Solve this for n.

$$2(n + 7) = 22$$
$$2n + 14 = 22$$
$$2n + 14 - 14 = 22 - 14$$
$$2n = 8$$
$$n = 4$$

Thus the particular number I chose at the beginning was 4.

Check: Take 4, add 7, and the result is 11. Twice 11 is 22, so this checks.

Do you see what words in the problem led to needing the distributive law?

Recall Example 4 from Eq. 3.
Example 4. One sixth less than three times a number is equal to 7/12. What is the number?
Do you see how the statement of Example 17 above differs from that and how that difference shows in the equation?

Look back at the solution in the previous section if you have forgotten how to do that Example 4.