Commentary: July 1, 2008

In responding to some students' questions, I have been thinking more about how students learn and have come to the conclusion that I need to re-structure some of my Eq. lessons. I will not do that this semester, while you are in the middle of them. But I want to give you, my current students, the benefit of my new insights, if you wish.

Right now, my Eq. 4 supplement is an introduction to using the distributive property in equations. Now I think that I should separate this into dealing with these types of problems using positive coefficients and dealing with these types of problems using a mixture of positive and negative coefficients and using subtraction. I have decided this because I know that students find it confusing to have to deal with both positive and negative numbers (as in Chapter F4) and I also know that students find using the distributive property difficult.

I also know that very few students write all the steps in working one of these problems. And that's fine. In my examples, I write a lot of steps because that's the only way I have to convey to students what I am thinking. But students want to skip some of these steps when doing the problems. So I need to give you some examples to show you a good, fairly short, way to write your solutions.

So, what does that mean for you this semester? On the next two pages of this handout are fully-worked-out solutions to examples of using the distributive property in algebraic expressions when we have to use negative numbers. On the last page of this handout are briefer good solutions to those same examples.

We will be working with these types of problems throughout the rest of the course. In our textbook, this is introduced in Lessons 2.1 and 2.2. But the time you see it there, I expect you to be good at using the distributive property with both positive and negative numbers.

After doing Eq. 4, I expect students to be good at using the distributive property with positive coefficients and to have some idea of how to deal with negative coefficients, but to build up their skills in that as they go through the rest of the Eq. lessons and Lessons 2.1 and 2.2.

Eq 4 Additional Material

Refine your skills in dealing with subtracting an expression in parentheses.

Objectives:

1. Understand how to write the full set of steps clearly –that is, using the definition of subtraction as addition of the opposite and using the distributive law. That is illustrated in Examples 1-5 following.
2. Understand both of those so clearly that you can visualize the steps in your head without writing them down. This is important because most students simply won't be patient enough to write all these steps every time, so it is important to do this extra step. This is illustrated in the re-written solutions of Examples 1-5 on the last page here.

Eq. 4. Additional Material. Explanation and Examples

Expressions with a subtraction of something in parentheses are very difficult for some students to handle. The key idea needed is that you can just think of the parentheses as 1 times the expression and then handle the expression just as you would if it were a different negative number besides -1.

Example 1: Simplify –5 (–3x + 7)

Solution:
= –5 (–3x + 7)
= –5 * (–3x) + (–5)*7 Use the distributive law.
= + 15x + (–35)
= 15x – 35 It is also correct to give the final answer as 15x + (–35)

Example 2: Simplify – (–3x + 7)

Solution:
= – (–3x + 7)
= –1 (–3x + 7) Note that 1 times anything doesn't change the value.
= –1 (–3x) + (–1)*7 Use the distributive law.
= + 3x + (–7)
= 3x – 7 It is also correct to give the final answer as 3x + (–7)

Notice that the method used in Example 2 is exactly like that in Example 1 after we have inserted the multiplier of 1 before the parentheses. This is the way to think of problems like this.

What about subtracting some expression in parentheses from some other expression? The following examples illustrate how to subtract something in parentheses using the distributive law

with negative numbers in various places. Notice that in some cases there are two equally correct ways to write the final answer.

Example 3: Simplify 4 –5 (–3x + 7)

Solution: 4 – 5 (–3x + 7)
= 4 + (–5) (–3x + 7) Subtraction is addition of the opposite.
= 4 + (–5) * (–3x) + (–5)*7 Use the distributive law.
= 4 + 15x + (–35)
= 15x – 31 It is also correct to give the final answer as 15x + (–31)

Example 4: Simplify 4 – (–3x + 7)

Solution: 4 – (–3x + 7)
= 4 – 1 (–3x + 7)
= 4 + (–1) (–3x + 7) Subtraction is addition of the opposite.
= 4 + (–1) * (–3x) + (–1)*7 Use the distributive law.
= 4 + 3x + (–7)
= 3x – 3 It is also correct to give the final answer as 3x + (–3)

Example 5: Simplify 17 – (8x – 5)

Solution: 17 – (8x – 5)
= 17 – 1 (8x – 5)
= 17 + (–1) (8x – 5) Subtraction is addition of the opposite.
= 17 + (–1) * (8x) – (–1)*5 Use the distributive law.
= 17 – 8x + 5
= –8x + 22

Now that you have seen several of these with all the steps, so that you understand all the steps clearly, it is time to consider which of these steps are so clear to you that you don't need to write them down. Most students don't like writing lots of steps. The ONLY WAY to skip them is to see them in your head clearly enough that you don't need to write them.

In the following re-write of these examples, I give only the steps that I would write. I have left blanks where the steps I omitted are. Thins of the blanks as places where you need to "think" something, but don't necessarily need to write it.

Example 1: Simplify –5 (–3x + 7)

Rewritten solution:

Solution: –5 (–3x + 7)

Use the distributive law.

= + 15x + (–35)

It is also correct to give the final answer as 15x –35

Example 2: Simplify – (–3x + 7)

Rewritten solution:

Solution: – (–3x + 7)

Use the distributive law.

= + 3x + (–7)

It is also correct to give the final answer as 3x – 7

Example 3: Simplify 4 –5 (–3x + 7)

Rewritten solution:

Solution: 4 – 5 (–3x + 7)

Subtraction is addition of the opposite.
Use the distributive law.

= 4 + 15x + (–35)

= 15x – 31 It is also correct to give the final answer as 15x + (–31)

Example 4: Simplify 4 – (–3x + 7)

Rewritten solution:

Solution: 4 – (–3x + 7)

Subtraction is addition of the opposite.
Use the distributive law.

= 4 + 3x + (–7)

= 3x – 3 It is also correct to give the final answer as 3x + (–3)

Example 5: Simplify 17 – (8x – 5)

Rewritten solution:

Solution: 17 – (8x – 5)

Subtraction is addition of the opposite.
Use the distributive law.

= 17 – 8x + 5

= –8x + 22