Eq. 8. Objectives

1. OPTIONAL: Solve equations with fractional coefficients by multiplying through by the denominator to clear the fractions. Check the solutions. DO NOT use this method unless you are confident that you are using it correctly.
2. Understand that the cross-multiplying to solve proportions with variables that we learned in Topic 3 is a special case of multiplying through by the denominator.

In most algebra textbooks, you will see that when an equation includes fractions, you multiply both sides by the common denominator first to get rid of the fractions. That is a very nice method, but most students find it very difficult to carry out this method correctly.

Read the discussion immediately below and read example 1 carefully.

Explanation and Examples

Solving equations with fractions: We learned that we can multiply both sides of an equation by the same nonzero number to obtain an equivalent equation. Suppose an equation has fractions in it. If we multiply both sides of the equation by some appropriately large number, then we can produce an equivalent equation with no fractions. (Sometimes this is called "clearing the fractions.") Many students find this an attractive method to use. However, some students find it confusing to do the required multiplications correctly. **Do not use this method unless you are completely confident that you are doing the multiplications correctly**. Remember that it is not necessary to use this method. You can solve every linear equation with the methods we learned before this.

Example 1. Solve $\frac{x}{2}+\frac{3}{4}=\frac{7}{8}$

Solution: $\frac{x}{2}+\frac{3}{4}=\frac{7}{8}$

Notice that there are three denominators in this problem: 2, 4, and 8. The LCD of these is 8. So we choose to multiply through both sides by 8.

$$8\cdot\left(\frac{x}{2}+\frac{3}{4}\right)=8\cdot\frac{7}{8}$$

$8\cdot\frac{x}{2}+8\cdot\frac{3}{4}=8\cdot\frac{7}{8}$ Use the distributive law.

$\frac{8x}{2}+\frac{8\cdot 3}{4}=\frac{8\cdot 7}{8}$ Notice that each fraction here can be reduced . . .

$4x + 6 = 7$ with no fractions left, because we multiplied through by the LCD.

$4x + 6 - 6 = 7 - 6$ Subtract the same number from both sides.

$4x = 1$

$$\frac{4x}{4} = \frac{1}{4}$$ Divide both sides by the same nonzero number.

$$x = \frac{1}{4}$$

Did you notice in Example 1 how we got rid of the fractions and wound up solving an equation that just has whole numbers in it? That, of course, is the point of this method.

Now, copy the problem for Example 2 and try to do it yourself.

1. Find the common denominator of the fractions.
2. Write the step where you indicate that you will multiply both sides by it.
3. Multiply.
4. Now look at the solution and see if you did this correctly.
5. Read the rest of the solution.

Example 2. Solve $\frac{x}{5} + \frac{3x}{10} = 8$

Solution: Here the LCD is 10. Even though the right-hand side doesn't have a fraction, it is very important to multiply that side by the same number as the other side, in order to obtain an equivalent equation.

$$10 \cdot \left(\frac{x}{5} + \frac{3x}{10}\right) = 10 \cdot 8$$

$$10 \cdot \frac{x}{5} + 10 \cdot \frac{3x}{10} = 10 \cdot 8$$ Use the distributive law.

$$\frac{10 \cdot x}{5} + \frac{10 \cdot 3x}{10} = 80$$ Multiply. Notice that each fraction can be reduced ..

$$\frac{\overset{2}{\cancel{10}} \cdot x}{\cancel{5}} + \frac{\overset{1}{\cancel{10}} \cdot 3x}{\cancel{10}} = 80$$

$$2x + 3x = 80$$ with no fractions left, because we multiplied through by the LCD.

$$5x = 80$$ Combine like terms.

$$\frac{5x}{5} = \frac{80}{5}$$ Divide both sides by the same nonzero number.

$$x = 16$$

Example 3 illustrates how to solve one of the hardest problems for this technique. Read the solution very carefully.

Example 3. Solve $\frac{5}{6}\left(2x-\frac{1}{5}\right)=\frac{7}{12}$

Solution: $\frac{5}{6}\left(2x-\frac{1}{5}\right)=\frac{7}{12}$

Here the denominators are 6, 5, and 12, so the LCD is 60. We could immediately multiply through both sides by 60, but most students make mistakes doing that. It is MUCH BETTER to use the distributive law to eliminate the parentheses first and then multiply both sides of the resulting equation by 60.

$$\frac{5}{6}\cdot 2x-\frac{5}{6}\cdot\frac{1}{5}=\frac{7}{12}$$ Use the distributive law.

$$\frac{5\cdot 2x}{6}-\frac{5\cdot 1}{6}=\frac{7}{12}$$ Multiply.

$$60\cdot\left(\frac{5\cdot 2x}{6}-\frac{5\cdot 1}{6}\right)=60\cdot\frac{7}{12}$$ Now multiply both sides by the LCD.

$$60\cdot\frac{5\cdot 2x}{6}-60\cdot\frac{5\cdot 1}{6}=60\cdot\frac{7}{12}$$ Use the distributive law.

$$\frac{60\cdot 5\cdot 2x}{6}-\frac{60\cdot 5\cdot 1}{6}=\frac{60\cdot 7}{12}$$ Multiply.

$$\frac{\overset{10}{\cancel{60}}\cdot 5\cdot 2x}{\cancel{6}}-\frac{\overset{10}{\cancel{60}}\cdot 5\cdot 1}{\cancel{6}}=\frac{\overset{5}{\cancel{60}}\cdot 7}{\cancel{12}}$$ Reduce fractions.

$10x - 5 = 35$ Now we have an equation with no fractions.

$10x - 5 + 5 = 35 + 5$ Add the same number to both sides.

$10x = 40$

$\frac{10x}{10}=\frac{40}{10}$ Divide both sides by the same nonzero number.

$x = 4$ Simplify.

Sometimes textbooks will indicate that you should solve a problem like Example 3 by multiplying through by the common denominator first, before you use the distributive law. While it is possible to do that, I have found that students in MATD 0330 are likely to do that incorrectly.

I believe that you'll be more successful if you use the distributive law first, as indicated in the solution to Example 3.

Proportion equations:

"Cross-multiplying": Recall that when we solved proportions, those were equations and you were told to cross multiply. Look at the example below. On the left it is done by cross-multiplying and on the right by multiplying through by a common denominator. Notice that it is exactly the same. This demonstrates that **the method of cross-multiplying a proportion to solve it is just a special case of multiplying both sides by the common denominator**.

Example 4. Solve $\frac{4}{x} = \frac{12}{27}$

Method of cross-multiplying	Method of multiplying by the common denominator
$\frac{4}{x} = \frac{12}{27}$ $4 \cdot 27 = 12 \cdot x$ $\frac{4 \cdot 27}{12} = \frac{12 \cdot x}{12}$ $9 = x$	The common denominator is 27 times x. $\frac{4}{x} = \frac{12}{27}$ $27 \cdot x \cdot \frac{4}{x} = 27 \cdot x \cdot \frac{12}{27}$ Mult. both sides by denominator. $\frac{27 \cdot x \cdot 4}{x} = \frac{27 \cdot x \cdot 12}{27}$ $27 \cdot 4 = 12 \cdot x$ Notice that the denominators cancel. $\frac{27 \cdot 4}{12} = \frac{x \cdot 12}{12}$ $9 = x$ Reduce

Most math teachers are not happy with students learning a rule about "cross-multiplying" to solve equations. While it is very convenient, it does not lead to general understanding of what's going on.

It is good to understand the explanation above so that you see what's really going on when we use this technique.

In this course, you are welcome to continue to use the method of cross-multiplying to solve equations with proportions like this. But I hope you will notice how it is really using the "balance" when we solve equations.

Please read the following discussion. Understanding what you are doing is very important in mastering mathematics.

How do we find what number to multiply through by? The point is to multiply through by a number that will eliminate the fractions. So it has to be a number that every denominator will divide into and give a whole number. Recall Example 1. $\frac{x}{2}+\frac{3}{4}=\frac{7}{8}$

If we had chosen to multiply through both sides by 4, then, on the right-hand side, we would not have eliminated the fraction.

If we had chosen to multiply through by 5 (which we wouldn't), we would not have eliminated any fractions at all.

However, if we had chosen to multiply through by 16, we would have eliminated all of the fractions. So that would have been a fine number to use.

This illustrates that we don't really have to use the LCD here – we just must use a number that all the denominators divide into. So we could have just used the product of all the denominators in the problem. In this case, that's 64. You do that yourself. Write down this problem and multiply both sides by 64. Then compare your solution to that in Example 1. Do you get the same answer? What's different about the solution?

If you choose to remember this technique as "multiply both sides by the product of the denominators" that would be a correct method.

Is using this method truly optional in this course? YES. Now that you have finished reading this supplement, decide whether you will be able to reliably use this method correctly. If you cannot, or even if you are uncertain, DO NOT USE IT. You can solve all linear equations without ever using it. All teachers would rather that a student use a method that they understand and can use correctly rather than a shortcut that they cannot reliably use correctly