Homework: Make a column down the right side of your page, about 1/3 of the page, titled " Comments". USE THIS.

Logic:

Logic is a way to find what you actually already know that you may not be aware of.

That is, sometimes the things you already know imply that something else has to be true. But you may not yet have thought of that something else. Logic helps you see that quickly and notice that it follows from what you already know.

For examples of this, see Lesson 4, exercises 8 and 9 (page 77). In these problems, we assume that you know the two things above the line. The question is whether those imply the statement below the line. If so, we say that "the argument is valid." Look at 8 and 9. Do you see now that one of those arguments is valid and one isn't? Which is which and why?

So it is also the case that we can use logic to avoid some mistakes that people sometimes make about what they think they know, but that they don't actually know.

I don't expect you to be good at this yet – I just want you to see what the point is about what we are doing here.

What sorts of statements are appropriate for us to use in logical arguments?

Sometimes people get annoyed with / don't believe logic because they feel that they are being told they must agree to something that they don't agree with. Sometimes that is because of some missing assumptions, or missing information.

What is a statement? Our text says "A simple statement is a declarative sentence about one idea that is either true or false. We don't have to know which it is – true or false."

An example of a declarative sentence that I don't consider to be a simple statement: "The presence of television in a household is good for children." My problem with this is that "good for children" is very "squishy." If two people get into a conversation about this, the only productive thing they can do is to talk about some of the things about television that they think are positive influences and other things that are negative influences and then they can discuss the importance of each of those things in their own thinking. (Maybe some individuals can only think of negative influences but even they should admit that not everyone can only think of negative influences, and vice versa.) So this short declarative sentence is masquerading as one idea, but it is actually summarizing several ideas based on some assumptions about the meaning of "good" and those assumptions aren't clearly stated.

Another reason this sentence is rather "squishy" is that "the presence of television" isn't very clear. Does that mean a television the children can watch broadcast TV on? Or does it include a television that can only be used with videos or DVDs? So there's some relevant information that is missing.

Summary: For logic to give us a good conclusion, it relies on the statements included being "crisp" statements. And "crisp" statements are statements where the information needed to define the terms and the assumptions needed to understand them are clear to the people participating in the conversation.

The meaning of "and" and "or":

Meaning of "and" Easy. Both of the statements are true.

Meaning of "or": Not so clear. Does it mean that only one of the statements is true or that at least one of the statements is true?

Example 1. Here are some statements from real life that are pretty clear.

- You will have homework in your marketing class on either Monday or Wednesday.
- You will have homework in your marketing class on Monday or Wednesday, or possibly both.

To have word to describe these, we could say "exclusive or" and "inclusive or". Which is which?

In real life, if your teacher says "You will have homework in class on Monday or Wednesday" which of those two statements above does she mean? Is it clear? What would you understand it to mean?

So we might say that "or" can mean two different things and which one depends on the context. That's certainly not very satisfactory for a mathematical logic system. So, in mathematical logic, they had to choose one of these for the meaning of "or." They chose the "inclusive or." So, in mathematics, "or" means one or the other or possibly both.

Why? Well, many things in the system are easier and simpler when we use the inclusive or. For instance, the negation of the statement "A and B" is "not A or not B" if we use the inclusive or. Think about that with the following example.

Example 2: Think of these two simple statements.

A: Jane is a high school graduate.

B: Jane is an assistant manager at Taco Cabana.

Write the statement "A and B" using the statements about Jane. Then write exactly what it would take to make the statement "A and B" false. That is the negation of the statement "A and B." (I think that this example shows that we sometimes think of "or" as the "inclusive or" in everyday language.)

Section 1 has many definitions and symbols. We'll see the important ones as we do some examples. Review this section as you do the assigned exercises. Take a few notes here.

Now let's look at some exercises in Section 1, pages 48-50:

Class: 2, 5, 6, 10, 11, 12, 22, 24, 28, 30, 42

Homework: 15, 17, 19, 21, 25, 29, 31, 33, 41, 43, 45, 47, 49, 51, 53

Truth Tables. The systematic way that mathematicians investigate whether statements are logically equivalent is with truth tables.

Example 3: **We are going to use truth tables to investigate how to write the negation of the statement "a and b" in order to see how it relates to "or."**

Here's a truth table for the statement "a and b".

a	**b**	**a and b**
T	T	T
T	F	F
F	T	F
F	F	F

Notice that in the first two columns, we put a **systematic** listing of **all** the possibilities for whether statements a and b are true or false.

In the third column, we list whether the compound statement "a and b" is true or false for the given values of the truth of the simple statements.

A negation of a statement means that True and False are reversed. So this is a truth table for the negation of the statement "a and b."

a	**b**	**a and b**	**~(a and b)**
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

Now, earlier I said that we use the "inclusive or" in math for several reasons, but one of which has to do with the negation of "a and b."

Let's investigate the statement "not a or not b" where we understand this to be the inclusive or.

a	**b**	**~a**	**~b**	**~a or ~b**
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Notice that, if we had used the "exclusive or" then the last line of the table would have given us a False and so it would not have matched the negation of the compound statement "a and b."

Example 4: Now we will investigate the negation of the compound statement "a or b".

a	**b**	**a or b**	**~(a or b)**
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Now let's look at the statement "not a and not b"

a	**b**	**~a**	**~b**	**~a and ~b**
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

So we can see from these truth tables that the negative of the statement “a or b” is logically equivalent to “not a and not b”

Examples 3 and 4 are summarized as “DeMorgan’s Laws” in Section 2, on page 57.

Example 5: What does the double negative mean?

In logic, where any statement is either true or false, then “not true” must mean false. So then “not false” must mean true.

a	**not a**	**not (not a)**
T	F	T
F	T	F

So “not (not a)” is logically equivalent to “a”.

I WILL NOT ask you to make truth tables. In the text, they make truth tables somewhat differently than I did here. The point of truth tables is to use symbols to clarify the meaning. I suspect that most students in this class find it more useful and easier to understand if we discuss the meaning rather than how to manipulate the symbols. Also, in the text, they are working up to looking at more complex statements than we will do. Truth tables are particularly useful for dissecting complex statements such as those in problems 28-34 and 52-55.

I want you to learn the basic ideas and deal with the less complex statements.

Section 2. Pages 59-61

Class: 37, 41, 43, 48, 63

HW: 36, 38, 39, 40, 42, 44, 46, 48, 62, 64

What does logic tell us about conditional statements?

Main result: When we have a **conditional statement "if p, then q"** then

- **The converse, "if q, then p" may or may not be true.** The statement "if p, then q" gives us no information about whether the converse is true.
- T**he contrapositive, "if not q, then not p" must be true.** This is logically equivalent to the statement "if p, then q."
- The inverse, "if not p, then not q" may or may not be true. (I include this for completeness. I have not seen this discussed as much as the others in real-world applications. This statement is logically equivalent to the converse.)

I know that our textbook does a lot with truth tables here, but I find that less than satisfactory for several reasons. I think it is good to just think about these and do some examples, until you understand WHY this result is true: **A conditional statement is logically equivalent to its contrapositive.**

Exercise 5: Here is a conditional statement that is true about me.

"If it is raining when I leave home in the morning, I will carry an umbrella."

Write the converse.

Write the contrapositive.

Write the inverse.

Which of those is logically equivalent to the original statement? (Hint: There is only one of these which is equivalent.)

At this point, we have gone over the main material in Section 3.

Section 3: pages 68-70

Class: 28, 32, 46, 48

HW: 27, 29, 31, 33, 45, 47, 49, 51, 53, 55, 57, 59

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Valid Arguments and Invalid Arguments

Read the four forms of valid arguments and the two forms of invalid arguments on page 74. Take notes that will help you remember what these mean. (Either copy the symbols or make up an example, or both.)

Valid: Law of Detachment

Valid: Law of Contrapositive

Valid: Law of Syllogism

Valid: Disjoint Syllogism

INVALID: Fallacy of the Converse

INVALID: Fallacy of the Inverse

Section 4: pages 76-79

Class: 8, 10

HW: 17-23 all odd-numbered problems.