Simple Harmonic Motion

The position of a point oscillating about an equilibrium position at time $t$ is modeled by either

$$s(t) = a \cos \omega t \quad \text{or} \quad s(t) = a \sin \omega t,$$

where $a$ and $\omega$ are constants, with $\omega > 0$. The amplitude of the motion is $|a|$, the period is $\frac{2\pi}{\omega}$, and the frequency is $\frac{\omega}{2\pi}$.

Rectangular and Polar Coordinates

If a point has rectangular coordinates $(x, y)$ and polar coordinates $(r, \theta)$, then these coordinates are related as follows:

$$x = r \cos \theta \quad y = r \sin \theta$$
$$r^2 = x^2 + y^2 \quad \tan \theta = \frac{y}{x}, \quad \text{if } x \neq 0$$

Applications of Parametric Equations

Parametric equations are used to simulate motion. If a ball is thrown with a velocity of $v$ feet per second at an angle $\theta$ with the horizontal, its flight can be modeled by the parametric equations

$$x = (v \cos \theta) t \quad \text{and} \quad y = (v \sin \theta) t - 16t^2 + h,$$

where $t$ is in seconds and $h$ is the ball's initial height in feet above the ground. The term $-16t^2$ occurs because gravity is pulling downward. See Figure 38. These equations ignore air resistance.

Looking Ahead to Calculus

At any time $t$, the velocity of an object is given by the vector $\mathbf{v} = (f'(t), g'(t))$. The object's speed at time $t$ is

$$|\mathbf{v}| = \sqrt{(f'(t))^2 + (g'(t))^2}.$$