UNDERSTAND derivations of 2-17 in this list.   MEMORIZE the results of 1-13 in this list.   Be able to USE all of 1-17 in this list.

How to Derive / Remember Trig Identities and Relationships

1. (Chapters 1 and 2) **Definitions**: MEMORIZE		2. (Ch. 1) **The Pythagorean identities**: Use $x^2 + y^2 = r^2$ and divide through by one of these terms to get one of the identities below. Then divide through by a different term to get a different identity below.
$\sin\theta = \frac{y}{r} = \frac{opposite}{hypotenuse}$ $\cos\theta = \frac{x}{r} = \frac{adjacent}{hypotenuse}$ $\tan\theta = \frac{y}{x} = \frac{opposite}{adjacent}$	$\csc\theta = \frac{r}{y} = \frac{1}{\sin\theta}$ $\sec\theta = \frac{r}{x} = \frac{1}{\cos\theta}$ $\cot\theta = \frac{x}{y} = \frac{1}{\tan\theta}$	$\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$

3. (Ch. 1) **Relationships:** Use the definitions above and simplify the fractions. $\tan\theta = \frac{\sin\theta}{\cos\theta}$; $\cot\theta = \frac{\cos\theta}{\sin\theta}$

4. (Chapter 2) **Cofunction Identities** Use the complementary angles in a right triangle and the definitions of the six trig functions in terms of *x*, *y*, and *r*, to derive these.

$\cos(90^0 - \theta) = \sin\theta$ $\sin(90^0 - \theta) = \cos\theta$ $\tan(90^0 - \theta) = \cot\theta$	$\cot(90^0 - \theta) = \tan\theta$ $\sec(90^0 - \theta) = \csc\theta$ $\csc(90^0 - \theta) = \sec\theta$

5. (Chapters 2 and 3) The **numerical values** of the sine, cosine, and tangent of those angles (whether the angle is given in degrees or radians): $\left\{0^0, 30^0, 45^0, 60^0, 90^0, 180^0, 270^0\right\}$ or $\left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}$

Use the coordinate system and the *x* and *y* coordinates and the definitions of trig functions to find $\left\{0^0, 90^0, 180^0, 270^0\right\}$ Use the Pythagorean Theorem to find a set of sides for these two right triangles: 30^0-60^0-90^0- and 45^0-45^0-90^0. Then use those lengths and the basic definitions of the trig functions to get the numerical values of these functions at $\left\{30^0, 45^0, 60^0\right\}$.

6. (Chapter 1 supplement and Chapter 4.) The **graphs** of the six trig functions and their domains and ranges. Plot points to construct the sine, cosine and tangent graphs and then MEMORIZE these graphs.
For the cosecant, secant, and cotangent graphs, use the reciprocal identities and your understanding of graphs of reciprocals to construct them and notice which points are not in the domains.

7. (Chapter 3) **The arc length formula** and area and velocity formulas derived from it.
One Radian = angle which intercepts a length on the circle equal to the radius of the circle.
Assume the angle θ is measured in radians. Derive the four formulas below from that definition of radian.

Arc length $s = r\theta$	Area of a sector: $A = \frac{\theta}{2\pi}\pi r^2 = \frac{1}{2}\theta r^2$	
Velocity $v = \frac{distance}{time}$	Angular velocity $\omega = \frac{angle}{time} = \frac{\theta}{t}$	Linear velocity: $v = \frac{arc\ length}{time} = \frac{r\theta}{t} = r\omega$

8. (Chapter 5) **Negative-angle formulas**: Use the unit circle and the *x* and *y* coordinates with the basic definitions of the trig functions and the symmetry around the positive *x*-axis of angles *A* and –*A*. Alternatively, use the basic graphs on the interval $[-360^0, 360^0]$ to see these and remember these.

$\sin(-A) = -\sin A$ $\cos(-A) = \cos A$ $\tan(-A) = -\tan A$	$\csc(-A) = -\csc A$ $\sec(-A) = \sec A$ $\cot(-A) = -\cot A$

9. (Chapter 5) **The double-angle formulas**: Use the sum formulas with both *A* and *B* the same. For the extra cosine double-angle formulas, start with the first cosine double-angle formula and use the Pythagorean identity to change either $\sin^2 A$ or $\cos^2 A$ to the other.

$\sin(2A) = 2\sin A \cdot \cos A$	$\cos(2A) = \cos^2 A - \sin^2 A$ $\cos(2A) = 2\cos^2 A - 1$ $\cos(2A) = 1 - 2\sin^2 A$

10. (Chapter 5) **The sum and difference formulas**:
1. Use a triangle/graphical argument to find $\cos(A-B)$.
2. Use the formula for $\cos(A-B)$ and plug in $-B$ to find a formula for $\cos(A+B)$.
3. To find $\sin(A+B)$, use the first co-function identity and the formula for $\cos(A+B)$.
4. To find $\sin(A-B)$, use the formula for $\sin(A+B)$ and plug in $-B$.

$$\boxed{\begin{aligned} \sin(A \pm B) &= \sin A \cdot \cos B \pm \sin B \cdot \cos A \\ \cos(A \pm B) &= \cos A \cdot \cos B \mp \sin A \cdot \sin B \end{aligned}}$$

11. (Chapter 7) **The Law of Sines**: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.
For a non-right triangle, construct a line from the vertex perpendicular to the opposite side. Call this line *h*. Use the two resulting triangles to find *h* in terms of the sine of the two other angles of the triangle. Follow this process for another side of the triangle to find the third part. (The law of sines also works for right triangles.)

12. (Chapter 7) **The Law of Cosines**: $a^2 = b^2 + c^2 - 2bc\cos A$.
Superimpose an *x*-*y* coordinate system appropriately on a non-right triangle and use the distance formula. (The law of cosines also works for right triangles.)

13. (Chapter 6) **The graphs of the three main inverse trig functions and their domains and ranges**.
Take the basic trig graph and restrict the domain so that it has an inverse function (is a one-to-one function.) Recall that inverse functions reverse the roles of *y* and *x*, and that means reflecting the graph across the line $y = x$ and interchanging the domain and range. Do that to find the domain, range, and graph of the inverse trig function.

14. (Chapter 5) **Tangent formulas:** sum, difference, and double angle.

Write each left-hand side in terms of sine and cosine of the sum, difference, or double angle. Then divide both numerator and denominator by the term that will give the 1 in the correct place in the result.

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$	$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$	$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

15. (Chapter 5) **Half-angle formulas:**

Take the cosine double angle formula with cosine on both sides. Solve for the cosine of the single angle. Rewrite it in terms of $A/2$ and *A* rather than *A* and *2A*. $\cos\frac{A}{2}=\pm\sqrt{\frac{1+\cos A}{2}}$ Take the cosine double angle formula with sine on the other sides. Solve for the sine of the single angle. Rewrite it in terms of $A/2$ and *A* rather than *A* and *2A*. $\sin\frac{A}{2}=\pm\sqrt{\frac{1-\cos A}{2}}$	Rewrite the left-hand side in terms of sine and cosine of $A/2$. Simplify. $\tan\frac{A}{2}=\pm\sqrt{\frac{1-\cos A}{1+\cos A}}$	Take the previous formula for $\tan\left(A/2\right)$ and rationalize the denominator. Verify the sign. $\tan\frac{A}{2}=\frac{\sin A}{1+\cos A}$ Take the previous formula for $\tan\left(A/2\right)$ and multiply the denominator by $1-\cos A$. $\tan\frac{A}{2}=\frac{1-\cos A}{\sin A}$

16. (Chapter 5) **Product-to-Sum Identities .** Start with the right-hand side of each of these identities and use the sum/difference formulas and simplify.

$\cos A\cos B=\frac{1}{2}\left[\cos(A+B)+\cos(A-B)\right]$	$\sin A\cos B=\frac{1}{2}\left[\sin(A+B)+\sin(A-B)\right]$
$\sin A\sin B=\frac{1}{2}\left[\cos(A-B)+\cos(A+B)\right]$	$\cos A\sin B=\frac{1}{2}\left[\sin(A+B)-\sin(A-B)\right]$

17. (Chapter 5) **Sum-to-Product Identities.** Start with a product-to-sum identity, make an appropriate substitution, solve for the new variables, and substitute. Here is one:

$\sin\alpha\cos\beta=\frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$. Let $\alpha+\beta=x$, $\alpha-\beta=y$. Then solve to find $\alpha=\frac{x+y}{2}$, $\beta=\frac{x-y}{2}$ and plug those into the product-to-sum identity to obtain the first sum-to-product identity below.

$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$	$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$
$\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$	$\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$