## Algebra with Trig Functions

One of the most crucial skills for using trigonometry in later math courses is the ability to correctly perform algebraic operations on trig functions.   Many students need to practice these types of problems about three or four at a time, with feedback from a tutor or teacher after each set and before doing more. We will start now so that all of you will be skilled at it when it is needed in chapter 5.

Consider the problem:  If  $\theta$  is in Quadrant IV and $\cos\theta = 0.72$, find  $\sin\theta$.
In Chapter 1, section 4, examples 4 and 5, we noticed that one method of solving this is:

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta + (0.720)^2 = 1$$
$$\sin^2\theta = 1 - (0.720)^2$$
$$\sin\theta = \pm\sqrt{1 - (0.720)^2}$$
$$\sin\theta = \pm 0.694$$

Because  $\theta$  is in Quadrant IV, then the value of  $\sin\theta$ is negative.  So  $\sin\theta = -0.694$.

**Example 1**:   Find an expression for  $\sin\theta$  in terms of  $\cos\theta$.
Solution:  We will solve it in a similar manner to the previous problem.

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta = 1 - \cos^2\theta$$
$$\sin\theta = \pm\sqrt{1 - \cos^2\theta}$$

**Example 2**:  Rewrite this expression in terms of  $\sin\theta$  and  $\cos\theta$  and simplify:  $\dfrac{1 + \tan^2\theta}{1 + \cot^2\theta}$.

$$\frac{1 + \tan^2\theta}{1 + \cot^2\theta} = \frac{1 + \dfrac{\sin^2\theta}{\cos^2\theta}}{1 + \dfrac{\cos^2\theta}{\sin^2\theta}}$$

$$= \frac{\dfrac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}}{\dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta}}$$

$$= \frac{\dfrac{1}{\cos^2\theta}}{\dfrac{1}{\sin^2\theta}} = \frac{1}{\cos^2\theta} \div \frac{1}{\sin^2\theta} = \frac{1}{\cos^2\theta} \cdot \frac{\sin^2\theta}{1}$$

$$= \frac{\sin^2\theta}{\cos^2\theta}$$

In one sense, this seems to be a fairly easy task because mostly the algebra used is fairly basic.   On the other hand, many students find it a bit confusing and need a considerable amount of practice in this.   We trig teachers would like to know how to predict which students will find this particularly difficult so that we can provide extra help.   But usually by the time we find out who is finding it difficult, there is not enough time for significant extra help.

In our textbook, this is introduced at the beginning of Chapter 5 and we begin to use it on many, many problems at that time.  In our class, we will start now and you should practice slowly on problems like these until you are confident that you can perform the algebra correctly. Working with problems like these will also give you important practice in remembering the trig identities.

When working problems like these, the most common mistake students make is to notice what would be a convenient simplification and not pay attention to whether it is algebraically correct.  As you do the following problems, pay careful attention to the algebraic steps.  It is more useful to do each step correctly and be unable to completely simplify the expression than to make an algebraic mistake at any step in the process.  Once you make an algebraic mistake, *no later step will be correct.*

1. Read p. 184-185, Chapter 5, Section 1, examples 2 and 3. Ask for help before going further if anything isn't clear.

2. Work Chapter 5, Section 1, problems 45, 46, 49, 51.  Check these and get feedback before going any further.

3. Work Chapter 5, Section 1, problems 47, 50, 55, 59.  Check these.  Notice that 47 and 59 are completely solved in the solutions at the end of the book.

4. If you had ANY difficulty with any of these problems, work more of the problems between 43 and 63 in this section.   Work no more than three of them at once and get feedback on them before going on.

If you found any of this difficult, then Chapter 5, Section 2 will also be difficult for you. It is an extremely important material in the course and if you need more time on it, you should start early so that you can take plenty of time.   To work the problems in Chapter 5, Section 2 you only need to know the trig identities and use algebra.   So you are ready to do it now.  Start doing problems 1-68  about three or four at a time and get help from a tutor or a teacher before you go on to the next three or four problems.   You are not required to do all of them – you are only required to do enough of them of each type to feel confident that you can do similar problems on a test.

Material from Chapter 5, Sections 1 and 2 will NOT be on Test 2.  It will be a major part of Test 3 and also a major part of the make-up test for this class.