

How to Answer Test 4 Questions

Overview

The problems in chapters 11-21 require students to all of these.

1. Recognize which technique / formula is appropriate.
2. Determine whether the conditions to apply that technique are met.
3. Set up the solution appropriately, including defining all parameters, and carry out the computations, with appropriate labels and diagrams.
4. Write conclusions that communicate an appropriate answer to the question in the problem. (For example, saying “Reject H_0 ” is not a complete conclusion, because it does not talk about the specific question in the problem.)

Most students focus on part 3 here as they are doing the homework, and don't practice the other parts enough. The examples on the next two pages demonstrate these steps. Before you take Test 4, it is crucial that you practice all the parts until you are able to do them well. Students who do poorly on Test 4 have not mastered all the important material in the course and almost always do poorly on Test 5 as well. Don't put yourself in that situation. Don't take Test 4 until you have mastered all these parts for the problems in Chapters 11-21.

Discussion of recognizing the appropriate technique

If you fully mastered the four-step process illustrated in our textbook, then you have a good start on this entire process. However, as you went through the chapters, you had almost no practice on the first part listed here – recognizing which technique is appropriate. As you do the homework, you probably just used whatever technique was in the chapter you just worked on. On the test, the problems won't be identified by the number of the chapter, so you'll have to determine the technique from the statement of the problem. Be sure to practice that quite a lot before you take the test.

The order of the parts of the questions

Obviously, a person should consider whether the conditions for a technique are met before actually carrying out the technique. I expect you to do that in this course, just as you will when you approach statistics problems in the real world.

However, testing and grading in a class brings up some additional considerations. These problems are rather long. Suppose one of these problems is worth 20 points on a test. Suppose the first part of the question is whether the conditions for a particular technique are met. Then suppose a student incorrectly determines that the conditions are not met and answers that, and then does not do any more work on the problem. That student would miss the entire problem – all 20 points worth. That seems like a bad thing.

It seems like a better idea to set up the test so that single mistakes are not likely to cost students 20 points. So, in this course, on a test over inferential statistics, for most of the questions, you will be asked to perform the most appropriate technique (of those we learned) first and then, after that, you will be asked to discuss what conditions were necessary for this

technique to be adequate and whether they were met. See the example problems on the next pages.

However, the department has decided that we must ask at least one question using the “four-step process.” So I will be doing that. On those problems, as well, even if you decide that the conditions aren’t met, for full credit, you must complete the problem by carrying out the appropriate inferential technique correctly. In your conclusion, you can give the usual type of conclusion and then another comment, if you wish, about whether you trust these results.

Interpretations / Conclusions

In every test problem where you are expected to find a confidence interval or do a hypothesis test, you will also be expected to interpret the result in terms of the question in the problem. This means that you must mention the variable in the problem, not just say “Reject H_0 .” I know that it is tedious to write all of those interpretations, since they are all so similar. But these are a very important part of the course and they are difficult for many students. Be sure to write enough interpretations on the quizzes and homework, and get feedback on them, so that you will be able to do this on the test.

Example A. To gather data on the full battery life of an iPod, researchers charged a battery, then used it to play an assortment of music until the battery ran out, and recorded the time in hours. In 48 trials they found $\bar{X} = 9.46$ and $s = 2.061$ and give the full battery life, between charges, for an iPod.

1. Is there significant evidence, at the 1% level, that the average full battery life for an iPod is less than 10 hours?
2. What conditions about the sampling method / experimental design are necessary to use the technique you used in part a?
3. What conditions about the distribution of the population are necessary to use the technique you used in part a?
4. Is there any reason to doubt that the conditions for this technique are met? What questions would you want to ask of the researchers to determine whether the conditions are met?

Solution:

This situation is about analyzing a mean rather than a proportion. It is about one population and the population standard deviation is unknown, but estimated by s . Thus we use the t-distribution.

1. $H_0 : \mu = 10$
 $H_a : \mu < 10$

where

$\mu =$ the mean life of a battery between charges for the entire population of iPods.

The sampling distribution of the test statistic when H_0 is true, has a t-distribution with 47

degrees of freedom, and mean 10 and standard deviation of $\frac{2.061}{\sqrt{48}} = 0.2975$

$$t = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{9.46 - 10}{\frac{2.061}{\sqrt{48}}} = -1.8152$$

Since $df = 47$ is not in the table, we use either the “conservative” approach of the closest one that is smaller, so that is $df=40$, or we can just use the closest df in the table, which is $df=5$. Either way, we find that the p-value of these data is between 0.025 and 0.05.

At the 1% level, do not reject H_0 because the p-value is not less than or equal to 0.01.

These data do not provide significant evidence, at the 1% level, that the population mean lifetime between charges of an iPod is less than 10 hours.

2. The data must be collected from a simple random sample.
3. Because $n > 40$, it is not necessary to assume anything in particular about the shape of the distribution of the population of hours of lifetime of iPod batteries between charges.
4. They don't describe how they got the sample data so we can't be sure it is a SRS. If they used just one battery and repeatedly charged it and ran it down, that would not be adequate to test the population mean lifetime for all batteries. So I'd ask whether they used just one battery and tested it repeatedly or whether they used multiple batteries.

Example B. In both 1996 and in 1999, the Gallup organization surveyed US adults and asked “Do you think there is life of some form on other planets in the universe or not? “ In 1999, out of 535 individuals surveyed, 326 responded yes. In 1996 out of 527 individuals surveyed 383 responded yes.

1. Form and interpret a 99% confidence interval for the difference in the population proportions of US adults who believe there is life on other planets between 1996 and 1999.
2. What conditions are necessary to use the technique you used in part a?
3. For each condition, discuss whether it is reasonable to assume that it was met.

Solution: The data in this problem are “yes/no” data, where the summary statistics are proportions, not means. Also, there are two proportions here, so we use the two-proportion techniques.

p_{99} = the population proportion of US adults in 1999 who believe there is life on other planets.

p_{96} = the population proportion of US adults in 1996 who believe there is life on other planets.

Estimate $p_{96} - p_{99}$ with a confidence interval.

$$\hat{p}_{99} = \frac{326}{535} = 0.6093 \quad \text{and} \quad \hat{p}_{96} = \frac{383}{527} = 0.7268, \text{ so}$$

$$SE = \sqrt{\frac{0.7268(1-0.7268)}{527} + \frac{0.6093(1-0.6093)}{535}} = \sqrt{0.000376777 + 0.000444959}$$

$$= \sqrt{0.000821729} = 0.02866583$$

$$(\hat{p}_{96} - \hat{p}_{99}) \pm z \cdot SE$$

$$(0.7268 - 0.6093) \pm 2.576 \cdot SE$$

The confidence interval is $0.1175 \pm 2.576 \cdot 0.02867$

$$0.1175 \pm 0.0739$$

$$0.0436 \text{ to } 0.1914$$

I have 99% confidence that $p_{96} - p_{99}$ is between 0.0436 and 0.1914.

(Alternatively, I have 99% confidence that between 0.0436=4.36% and 0.1914=19.14% more of the population of US adults believed that there is life on other planets in 1996 than in 1999.)

2. These conditions are necessary to use this large-sample confidence interval.
 - Independent samples from the two populations
 - Simple random samples
 - In both samples, the numbers of both successes and failures are at least 10.
3. For the first condition, these are clearly independent samples. For the second condition, since this is the Gallup organization, which is well-known for scientific sampling, it seems reasonable to assume that these are simple random samples. For the third condition, one sample had 326 successes and 209 failures and the other sample had 383 successes and 144 failures. All those numbers are at least 10.