Statistics Resources in a “Math for Practical Arts” Course

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Math for Measurement

For whom: A general education math course designed to appeal to students in technical areas such as welding, building trades, physical therapy and other areas where a standard college algebra course is not the most useful math course for the students.

Prerequisite: Passed Texas' THEA test or equivalent to be minimally eligible for college-level math. (Approximately equivalent to having demonstrated skill in Elementary Algebra.)

Goals:

- Improve students’ ability to think with mathematics.
- Increase students’ belief that mathematics they can use is applicable to real-world problems.
- Prepare the students, as future technicians, to participate in technical conversations with engineers in their field.

Overall "Strands" in the course:

- Applied trigonometry
- Error propagation in calculations when using approximate numbers
- Modeling using linear, quadratic, exponential and other mathematical functions

Pedagogical style:

Discussion and problem solving in a "spiraling" approach. In each four-week quarter of the course, some types of problems from each of the three themes are introduced, building on the ideas learned in the previous quarter.
The webpage has some "flow charts" to help you follow a theme through the course. (These will be uploaded later today.)

Web Resources available:

- Complete textbook, in 27 topics.
- Daily handouts used in Fall 2008, which give an overview of material covered in each class period.
- Spreadsheets set up to generate random data for each of the types of relationships used here. You can specify the parameter values and how "noisy" you want the data to be. (Will be uploaded later today)

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Modeling:

We teach students to create a spreadsheet to fit a formula to data using the least squares criteria.

Our main functions are linear, quadratic, and exponential.

They also learn to make some modifications.

(Go to the Excel workbook for an example.)

- Use natural parameters, so they are easy to interpret.
  Quadratic: $y = a(x-h)^2 + k$

- Give students some strategies so that fitting by hand is not just “trial and error.”

- Assess whether one type of model fits better than another by (1) graphs, (2) pattern (or lack of pattern) of positive and negative residuals, and (3) size of standard deviation.

- We discuss interpolation and extrapolation, and the relative importance of getting the “right” type of model for each.
- Modify fitting criteria: minimize maximum squared deviation, minimize sum of squared relative deviations.
- Omit an outlier from fitting while still leaving on the graph (by zeroing out the squared deviation entry for it.)

- Use semi-log and log-log graphs to distinguish between exponential and power relationships. (Introduce logs as part of this topic. I’m amazed at how quickly we can do this!)
- At the end we mention (lightly) more advanced functions to explore: Normal, logistic, sinusoidal, logarithmic. These are covered fairly fully in the materials, but we don’t expect students to work with them in the course.

Error propagation

- Approximate numbers can be either rounded numbers or "noisy" numbers. Noisy numbers are those from a measurement process.
- It is easier for students to understand error propagation on rounded numbers because of the crisp endpoints, so that's where we begin. The same kinds of ideas apply to noisy numbers and we lead the students through those as well.
- When we introduce "noisy" numbers, we model the measurement process by a normal distribution and teach students to **estimate** the standard deviation and **use a spreadsheet to compute** the standard deviation.

Example 2. **Error propagation on rounded numbers**

The angle from the ground to the top of a tower is 70°, rounded to the nearest degree, and the point on the ground is 32 feet from the base of the tower, rounded to the nearest foot. How tall is the tower? Give the result (a) in significant digits and (b) using the method of error propagation and (c) discuss how these answers differ and what gives the most accurate description of reality.

$a = b \cdot \tan A$ $a = 32 \text{ ft} \cdot \tan 70^\circ$ $a = 87.91927742 \text{ ft}$	(a) Significant digits 88 feet which implies that the height is between 87.5 and 88.5 feet.

(b) The range of actual values for angle *A* is 69.5˚ to 70.5˚. The range of actual values for side *b* is 31.5 ft to 32.5 ft. Use the formula four times to compute the length of side *a* for four different sets of measurements.

	A = 69.5°	A = 70.5°
b = 31.5 ft	Computed a = 84.25	Computed a = 88.95
b = 32.5 ft	Computed a = 86.93	Computed a = 91.78

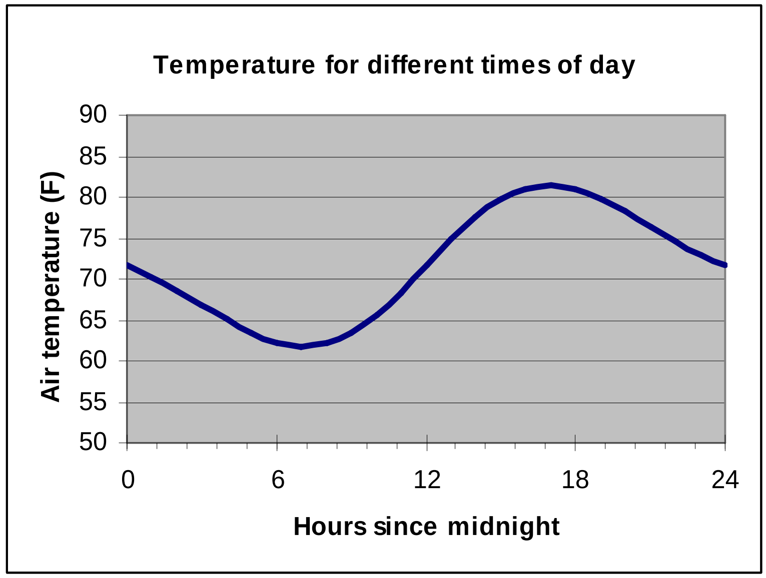
Thus, the length of side *a* is between 84.25 ft and 91.78 ft.

(c) The result from the error propagation method is based on the actual precision of the given measured angle and length and so is the correct answer to the question of what interval of actual values is possible for the computed length, given the measured angle and length.

The result from the method of significant digits implies an unrealistic precision of the computed length, but at least it does imply some imprecision, and is definitely preferable to the “math class” answer of 87.9193 feet.

Example 3. Input sensitivity

For this graph, at what time would being 10 minutes late in measuring the temperature result in the largest overestimate?



(Relevant to either rounded numbers or "noisy" numbers.)

Being late will cause an overestimate where the temperature is increasing, so the answer must be during the period from about 8 am to 4 pm, where the graph is noticeably rising.

The biggest mistakes will happen where the slope of graph is steepest at about 12 hours after midnight, which is noon.