Topic B. Solving Equations and Evaluating Expressions and Checking your Work

Objectives:

1. Use additional resources to practice algebra, as needed.
2. Learn to check answers in some way other than looking "in the back of the book."
3. Use the distributive property and other properties to simplify algebraic expressions and check your work.
4. Solve and simplify linear equations and check solutions of equations by plugging them back in.
5. Solve equations with variables in the denominator and check solutions by plugging them in.
6. Solve equations of those two types with multiple variables for one variable, resulting in a formula for that one variable.
7. Evaluate variable expressions when given the value of each variable, using the correct order of operations.

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Overview

The prerequisite for these materials includes mastery of basic math skills and some algebra, as needed to achieve "college-level" on Texas' TSI test. Many students who have learned math at this level forget it easily, so this review section of some algebra is provided. Some students in the class will go through this material quickly and others will need to go much more slowly and do more review work. Additional optional practice problems are provided on the course web pages. The next few topics in the do not use this algebra, so it isn't necessary to master everything in this topic immediately.

A major difference between this course and most other math courses involves "checking your answer by looking in the back of the book." In standard algebra classes, students practice quite a lot of similar problems and are expected to check many of the answers by looking in the back of the book. When we use mathematics in applications in the real world, there is no "back of the book." In this course, we will learn methods for independently checking our results to see if they are correct, or at least reasonable, without having to rely on answers that someone else gives us. Most students find this awkward at first, but when they persist in practicing the checking methods, develop more confidence in using mathematics and in their problem-solving skills. Even when a solution is provided in the "back of the book" we expect you to pay attention to other ways of checking your work, and to use those frequently (including on test questions.)

Section 1. Use the distributive property and other properties to simplify expressions and check your work.

Distributive Property:
For any real numbers a, b, and c: $a(b + c) = ab + ac$.
For any real numbers a, b, and c: $a(b - c) = ab - ac$

The fact that two expressions are equal means that they are equal for any values of the variable. So you can check by taking a few values for the variable and making sure that those do make the two sides equal. Usually we don't use the values 0 or 1 or 2 as the value for the variable and it is best to avoid numbers that already appear in the problem.
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Example 1. Simplify $10(4x+5)$.

Solution:
$10(4x+5)=40x+50$

Partial Check: Use $x=3$

$$10(4x+5)=40x+50$$
$$10(4\cdot 3+5)\ ?=?\ 40\cdot 3+50$$
$$10(12+5)\ ?=?\ 120+50$$
$$10(17)\ ?=?\ 170$$
$$170 = 170$$

Example 2. Simplify $-6(x+9)$.

Solution:
$-6(x+9)=-6x-54$

Partial Check: Use $x=5$

$$-6(x+9)=-6x-54$$
$$-6(5+9)\ ?=?\ -6*5-54$$
$$-6(14)\ ?=?\ -30-54$$
$$-84=-84$$

Example 3. Simplify $2(3x-7)$.

Solution:
$$2(3x-7)=2\cdot 3x-2\cdot 7$$
$$=6x-14$$

Partial Check: Use $x=5$

$$2(3\cdot 5-7)\ ?=?\ 6\cdot 5-14$$
$$2(15-7)\ ?=?\ 30-14$$
$$2(8) = 16$$

Example 4. Simplify $17-5(4x-12)$.

Solution:
$$17-5(4x-12)=17-20x-(-60)$$
$$=17-20x+60$$
$$=77-20x$$

Partial Check: Use $x=3$

$$17-5(4\cdot 3-12)\ ?=?\ 77-20\cdot 3$$
$$17-5(12-12)\ ?=?\ 77-60$$
$$17-0 = 17$$

Caution: Checking your work on problems like these is not completely satisfactory, because students who make a mistake in the original solution often make a corresponding mistake in the checking process. So the fact that it checks does not provide complete confidence that the solution is correct. But it's still a good idea to check because you'll catch most of your mistakes.

For links to material with additional examples and explanations, see the course web page.

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Section 2. Solving and simplifying linear equations and checking the result.

To solve equations, write a simpler equivalent equation using the following rules. Keep writing simpler equivalent equations until you have simplified it enough that one side has the variable alone and the other has a number. That gives the solution.

1. Add or subtract the same number or expression to both sides of the equation.
2. Multiply or divide both sides of the equation by the same nonzero number or expression.

You can check the solution to an equation by plugging the number into the original equation and seeing if the resulting statement is true.

For additional explanation and examples, see the course web page for links. These include links to instructional materials and a link to an equation solver, where you can enter any equation and have it find the solution. You can use that to check your work on as many problems as you have the time and energy to work.

Example 1. Solve $3x-4=11$.

Solution:

$$3x-4=11$$
$$3x-4+4=11+4$$
$$3x=15$$
$$\frac{3x}{3}=\frac{15}{3}$$
$$x=5$$

Check:

$$3x-4=11$$
$$3\cdot 5-4 ?=? 11$$
$$15-4 ?=? 11$$
$$11=11$$

Example 2. Find a formula for y (that is, solve for y in terms of x): $y-7=-4(x-2)+12$

Discussion: This is more complicated than a problem like Example 1. *However, it uses exactly the same techniques*. Problems like this arise when we begin to investigate graphing lines. Probably you have done problems like these in a previous algebra course, but maybe you haven't done many of them. We will practice many problems like these in the first few weeks of the course. Don't practice these until you can do problems like Example 1 fairly easily.

Solution:

$$y-7=-4(x-2)+12$$
$$y-7=-4x+8+12$$
$$y-7=-4x+20$$
$$y-7+7=-4x+20+7$$
$$y=-4x+27$$

Check by plugging in the answer for y:

$$y-7=-4(x-2)+12$$
$$(\quad)-7=-4(x-2)+12$$
$$(-4x+27)-7 ?=? -4(x-2)+12$$
$$-4x+27-7 ?=? -4x+8+12$$
$$-4x+20 = -4x+20$$

Example 3. Solve $0.75-0.08t=1.22$

Discussion: This is basically like Example 1, except that the coefficients are decimals and the variable is *t* rather than *x*. In your previous algebra classes, you may not have done much work with decimal numbers, but we will use them often in this class. We will practice many problems like these in the first few weeks of the course. Don't practice these until you can do problems like Example 1 fairly easily.

Solution:

$$0.75-0.08t=1.22$$
$$0.75-0.08t-0.75=1.22-0.75$$
$$-0.08t=0.47$$
$$\frac{-0.08t}{-0.08}=\frac{0.47}{-0.08}$$
$$t=-5.875$$

Check:

$$0.75-0.08t=1.22$$
$$0.75-0.08\cdot(-5.875)\ ?=?\ 1.22$$
$$0.75+0.47\ ?=?\ 1.22$$
$$1.22=1.22$$

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Section 3. Solving equations with variables in the denominator.

Example 1: Solve $\frac{14}{3}=\frac{8}{x}$

Discussion: Sometimes students learn to solve problems like this one by "cross-multiplying." That is correct. However, most math teachers prefer to think of solving this by multiplying both sides by the same thing – in this case the product of the two denominators. The result is the same. Both methods are shown below.

Solution:

Solution Method: *Cross-multiply:*	Alternate Solution Method: *Multiply by common denominator*	Check:
$\frac{14}{3}=\frac{8}{x}$ $14x=3\cdot 8$ $14x=24$ $\frac{14x}{14}=\frac{24}{14}$ $x=1.7143$	$\frac{14}{3}=\frac{8}{x}$ $3\cdot x\cdot\frac{14}{3}=3\cdot x\cdot\frac{8}{x}$ $\frac{3}{3}\cdot x\cdot 14=\frac{x}{x}\cdot 3\cdot 8$ $1\cdot x\cdot 14=1\cdot 3\cdot 8$ $14x=24$ $\frac{14x}{14}=\frac{24}{14}$ $x=1.7143$	$\frac{14}{3}=\frac{8}{x}$ $\frac{14}{3}\ ?=?\ \frac{8}{1.7143}$ $4.66667\ ?=?\ 4.66663$ Here, the two sides aren't exactly equal. But that isn't surprising, because we rounded the final answer to the original problem, so we don't expect the two sides here to be exactly equal.

Notice that the quotient, which is the final answer, didn't come out even, so I had to round off. I chose to round off to four decimal places. We'll talk more specifically about how exactly answers should be given later in the course. For now, when you have to round in the final computation, always keep at least three decimal places. That will ensure that your checking will produce answers close enough that you can recognize them as being essentially the same, so your checking is useful.
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Example 2: Solve $\frac{7}{33} = \frac{x}{5}$

Discussion: Here the variable isn't in the denominator, but this illustrates that the basic principle of multiplying both sides by the same non-zero expression works here too.

Solution:

Solution Method: *Cross-multiply:*	Alternate Solution Method: *Multiply by common denominator*	Check:
$\frac{7}{33} = \frac{x}{5}$ $33x = 7 \cdot 5$ $33x = 35$ $\frac{33x}{33} = \frac{35}{33}$ $x = 1.0606$	$\frac{7}{33} = \frac{x}{5}$ $33 \cdot 5 \cdot \frac{7}{33} = 33 \cdot 5 \cdot \frac{x}{5}$ $\frac{33}{33} \cdot 5 \cdot 7 = \frac{5}{5} \cdot 33 \cdot x$ $1 \cdot x \cdot 35 = 1 \cdot 33 \cdot x$ $33x = 35$ $\frac{33x}{33} = \frac{35}{33}$ $x = 1.0606$	$\frac{7}{33} = \frac{x}{5}$ $\frac{7}{33}\ ?=?\ \frac{1.0606}{5}$ $0.21212 = 0.21212$

Example 3: Solve $\frac{12}{x} = 6$

Solution Method: *Cross-multiply:*	Solution Method: *Multiply by common denominator*	Check:
$\frac{12}{x} = \frac{6}{1}$ $6x = 12$ $x = 12/6 = 2$	$\frac{12}{x} = 6$ $x \cdot \frac{12}{x} = x \cdot 6$ $\frac{x}{x} \cdot 12 = 6x$ $12 = 6x$ $x = 12/6 = 2$	$\frac{12}{x} = 6$ $\frac{12}{2} = 6$

Example 4: Find a formula for *h* (that is, solve for *h*.) $\frac{h}{36} = \frac{m}{k}$.

Solution Method: *Cross-multiply:*	Alternate Solution Method: *Multiply by common denominator*	Check:
$\frac{h}{36} = \frac{m}{k}$ $hk = 36m$ $\frac{hk}{k} = \frac{36m}{k}$ $h = \frac{36m}{k}$	$\frac{h}{36} = \frac{m}{k}$ $36k\frac{h}{36} = 36k\frac{m}{k}$ $hk = 36m$ $\frac{hk}{k} = \frac{36m}{k}$ $h = \frac{36m}{k}$	$\frac{h}{36} = \frac{m}{k}$ Substitute $\frac{\frac{36m}{k}}{36} =$ $\frac{36 \cdot m}{k} \div 36 =$ $\frac{36 \cdot m}{k} \cdot \frac{1}{36} =$ $= \frac{m}{k}$

Example 5: Find a formula for *d* (that is, solve for *d*.) $\frac{a}{0.37} = \frac{r}{d}$.

Solution Method: *Cross-multiply:*	Alternate Solution Method: *Multiply by common denominator*	Check:
$\frac{a}{0.37} = \frac{r}{d}$ $0.37r = a \cdot d$ $\frac{0.37r}{a} = \frac{a \cdot d}{a}$ $\frac{0.37r}{a} = d$ $d = \frac{0.37r}{a}$	$\frac{a}{0.37} = \frac{r}{d}$ $0.37d \cdot \frac{a}{0.37} = 0.37d \cdot \frac{r}{d}$ $d \cdot a = 0.37r$ $\frac{d \cdot a}{a} = \frac{0.37r}{a}$ $d = \frac{0.37r}{a}$	$\frac{a}{0.37} = \frac{r}{d}$ Substitute for d on the right. $\frac{r}{d} = \frac{r}{\frac{0.37r}{a}}$ $= r \div \frac{0.37r}{a}$ $= \frac{r}{1} \div \frac{0.37r}{a}$ $= \frac{r}{1} \cdot \frac{a}{0.37r}$ $= \frac{ra}{0.37r}$ $= \frac{a}{0.37}$

Section 4. Evaluate variable expressions when given the value of each variable, using the correct order of operations.

Recall these rules about order of operations. Do them in this order.

1. All operations inside symbols of grouping (parentheses) from the inside out.
2. All operations of exponents or roots.
3. All multiplications and divisions, in order from left to right.
4. All additions and subtractions, in order from left to right.

The best method I know is to write the expression first with the variable, then with parentheses in place of the variable, and then with the values inserted into the parentheses.
Then you can remove any of those parentheses that aren't needed to keep the negative numbers clear and to keep the products of two numbers clear.
Then, begin to evaluate the expression according to the order of operations, doing one operation per step.

For problems in this course, the most important of these rules are to do exponents first and then to do multiplications/divisions before additions/subtractions.

Example 1. Evaluate $y = 6 + 2x$ when $x = 7$.

Solution:

$$y = 6 + 2x$$
$$y = 6 + 2 \cdot (\ \ )$$
$$y = 6 + 2 \cdot (7)$$
$$y = 6 + 2 \cdot 7$$
$$y = 6 + 14$$
$$y = 20$$

Notice that the 7 here is in parentheses, but it is not an OPERATION in parentheses, so there is no need to think about that first rule in the order of operations.

Discussion: Later in the course, some students frequently do problems like this incorrectly because they do the addition before the multiplication. It is important to learn this rule well.

Example 2: Evaluate $y = ux^4$ when $x = 2$ *and* $u = 9$.

Solution:

$$y = ux^4$$
$$y = (\ \ ) \cdot (\ \ )^4$$
$$y = (9) \cdot (2)^4$$
$$y = 9 \cdot 2^4$$
$$y = 9 \cdot 16$$
$$y = 144$$

Discussion: Later in the course, some students have trouble remembering not to do the multiplication of 9 times 2 here first. Many of our formulas will include exponents such as this, so it is important to learn this rule well.

Checking your work: When evaluating expressions, the only clear method to check your work is to simply re-work it. That's not completely satisfactory because if you have a mistake in understanding, you're likely to make it both times.

However, in this course, most of the times when we will evaluate an expression, it is in the context of a larger problem for which there is a good method of checking available.

Exercises.

After you work each of these problems, use some method of checking your answer and show that check right beside your solution. Don't forget to prepare and fill out the homework cover sheet as you do the problems.

Part I.

1. Simplify $10(4x+5)$
2. Simplify $-6(x+9)$
3. Simplify $2(3x-7)$
4. Simplify $17-5(4x-12)$
5. Solve $3x-4=11$
6. Find a formula for y (that is, solve for *y* in terms of *x*): $y-7=-4(x-2)+12$
7. Solve $0.75-0.08t=1.22$
8. Solve $\frac{14}{3}=\frac{8}{x}$
9. Solve $\frac{7}{33}=\frac{x}{5}$
10. Solve $\frac{12}{x}=6$
11. : Find a formula for *h* (that is, solve for *h*.) $\frac{h}{36}=\frac{m}{k}$.
12. Find a formula for *d* (that is, solve for *d*.) $\frac{a}{0.37}=\frac{r}{d}$.
13. Evaluate $y=6+2x$ when $x=7$
14. Evaluate $y=ux^4$ when $x=2$ *and* $u=9$

Part II.

15. Solve $8x+7=31$
16. Solve $6x-14=40$
17. Simplify $13(x+2)$
18. Simplify $8(x-2)$
19. Solve $\frac{5}{2}=\frac{35}{x}$
20. Solve $\frac{9}{r}=\frac{54}{30}$
21. Solve $\frac{7}{5}=\frac{x}{35}$
22. Solve $17=-3x+2$
23. Find a formula for y (that is, solve for *y*): $y-7=4(x-3)$
24. Find a formula for y (that is, solve for *y*): $y-15=3(x-7)$
25. Find a formula for *k* (that is, solve for *k*.) $\frac{m}{2.1}=\frac{k}{t}$.
26. Find a formula for *m* (that is, solve for *m*.) $\frac{b}{m}=\frac{w}{17}$.
27. Evaluate $L=A\cdot r^t$ where $A=10,\ r=2,\ t=5$
28. Evaluate $y=7x-3$ where $x=5$