Topic C. Communicating the Precision of Measured Numbers

This topic includes
Section 1. Reporting measurements
Section 2. Rounding
Section 3. Precision of a number, including the concept of significant digits
Section 4. Identifying the interval of actual values for a rounded number.

Objectives:
1. Practice the usual rules for rounding.
2. Round a number to a given precision.
3. Determine the implied precision of a rounded number, giving it in words, or by a number, or by underlining the significant digits.
4. Understand why we don't do two sequential rounding operations on a number, but do all rounding in one step.
5. Understand that rounding numbers ending in exactly half is sometimes done in a different way.
6. On a number line, graph the interval of actual values which are consistent with a rounded value.
7. Use interval notation to give the interval of actual values which are consistent with a rounded value.
8. Use inequality notation to give the interval of actual values which are consistent with a rounded value.
9. For a given rounded number, find the maximum error due to rounding and express it as a percentage of the rounded number.

Section 1. Reporting Measurements

The following examples use the metric system for measuring lengths. The ideas are applicable to measurement in other contexts, such as volume or area, and using the English system as well as the metric system. These examples were chosen because it is easier to convey precision with decimals than fractions, and the metric system makes it easy to use decimals. Look at your ruler and notice that the centimeter is divided into tenths (each of those is called a millimeter) and the inch is divided into either eighths or sixteenths. So when we want to make measurements with a ruler smaller than one centimeter, we can easily use decimals, but when we want to make measurements smaller than one inch with a ruler, it is natural to use fractions rather than decimals.

Suppose we are using a ruler to measure the lengths of some pieces of cardboard and we give each measurement, along with a phrase describing how precisely we measured it.

Measurement		
12 cm, correct to the nearest cm.		
18.2 cm, correct to the nearest tenth of a cm.		
33 cm, correct to the nearest tenth of a cm.		
30 cm, correct to the nearest cm.		
Estimate of 20 cm, correct to the nearest ten cm.		
Estimate of 180 cm, correct to the nearest ten cm.		
Estimate of 300 cm, correct to the nearest ten cm.		
Estimate of 300 cm, correct to the nearest hundred cm.		

It would be more convenient to give each number in a way that conveys the precision instead of having to write the phrase afterwards. The generally accepted method for doing that is to report exactly the same number of digits as were observed. That is straightforward in many situations. It is somewhat less straightforward when the most precise digit observed is a "trailing zero." But that is also easily taken care of for the third number in our list, 33 cm, rounded to the nearest cm. Here we use a trailing zero after the decimal place. In arithmetic courses, we learned that when we compute with the numbers 33 and 33.0, we will obtain the same results. So if someone writes 33.0 instead of 33, they must be intending to convey something more than just the exact value of 33. They are conveying that the number is approximately 33 and also conveying the precision of that approximation.

The fourth number in our list, 30 cm, correct to the nearest cm, can be written as 30. cm. When we explicitly include a decimal we indicate that each of the digits before the decimal was measured precisely. I think of this as only a fairly clear report rather than a clear report, because a decimal point at the end of the number is so easily overlooked or forgotten in copying the number.

Measurement	**Completely clear correct report**	**Fairly clear correct report**
12 cm, correct to the nearest cm.	12 cm	
18.2 cm, correct to the nearest tenth of a cm.	18.2 cm	
33 cm, correct to the nearest tenth of a cm.	33.0 cm	
30 cm, correct to the nearest cm.		30. cm
Estimate of 20 cm, correct to the nearest ten cm.		
Estimate of 180 cm, correct to the nearest ten cm.		
Estimate of 300 cm, correct to the nearest ten cm.		
Estimate of 300 cm, correct to the nearest hundred cm.		

The next four numbers in our list are somewhat more difficult to report clearly without words. For a fairly clear correct report, we could say that we will assume that all trailing zeros in the number that would be needed to indicate the size are NOT to be interpreted as conveying precision. If we do that, then three of these four would be interpreted correctly.

Measurement	**Completely clear correct report**	**Fairly clear correct report**
12 cm, correct to the nearest cm.	12 cm	
2.7 cm, correct to the nearest tenth of a cm.	2.7 cm	
18.2 cm, correct to the nearest tenth of a cm.	18.2 cm	
33 cm, correct to the nearest tenth of a cm.	33.0 cm	
30 cm, correct to the nearest cm.		30. cm
Estimate of 20 cm, correct to the nearest ten cm.		20 cm
Estimate of 180 cm, correct to the nearest ten cm.		180 cm
Estimate of 300 cm, correct to the nearest ten cm.		
Estimate of 300 cm, correct to the nearest hundred cm.		300 cm

We see that there is an advantage in having the most-precise digit in the number after the decimal point when we want to report the number concisely and have the precision be clear. We could do that by changing our scale so that the relevant measurements are all decimals. In this case, since 100 centimeters is 1 meter, we could simply change our measurements to meters from centimeters and then use trailing zeros as needed to convey the precision completely clearly.

The conversion calculation needed for the first measurement in our table is

$$
\begin{aligned}
12\ \text{cm} &= \\
&= \frac{12\,\text{cm}}{1} \cdot \frac{1\ \text{m}}{100\,\text{cm}} \\
&= \frac{12 \cdot 1}{1 \cdot 100}\ \text{m} \\
&= 0.12\ \text{m}
\end{aligned}
$$

Review. It is important to be able to convert measurements between different types of units using the proportion method, as illustrated above. Notice that the proportion method enables you to keep track of the units algebraically. This is an important skill. See the course web pages for additional explanations and examples of this method of measurement conversion.

Measurement	**Completely clear correct report in centimeters**	**Completely clear correct report in meters**
12 cm, correct to the nearest cm.	12 cm	0.12 m
2.7 cm, correct to the nearest tenth of a cm.	2.7 cm	0.027 m
18.2 cm, correct to the nearest tenth of a cm.	18.2 cm	0.182 m
33 cm, correct to the nearest tenth of a cm.	33.0 cm	0.330 m
30 cm, correct to the nearest cm.		0.30 m
Estimate of 20 cm, correct to the nearest ten cm.		0.2 m
Estimate of 180 cm, correct to the nearest ten cm.		1.8 m
Estimate of 300 cm, correct to the nearest ten cm.		3.0 m
Estimate of 300 cm, correct to the nearest hundred cm.		3 m

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Section 2. Rounding.

Suppose we want to make a graph to summarize the heights of a class of 50 people. And we have their heights measured to the nearest tenth of an inch. The purpose of making this graph is to get a feeling for the variability of their heights. So the numbers as measured are more accurate than we really need – heights to the nearest inch would be adequate and easier to handle. Here is a portion of the dataset.

Original	61.3	68.5	71.4	65.8	64.3	63.4	67.2	72.3	69.5	70.1	62.8	63.7	65.2
Rounded	61	69	71	66	64	63	67	72	70	70	63	64	65

The usual rule for rounding is that, when the part you will drop is <u>less</u> than half, you "go down" and when the part you will drop is <u>equal to or more</u> than half, you "go up."

In a later section in this Topic, we will also learn to "think backwards" to see the interval of actual values that are consistent with a particular rounded value.

Example 1. Round to the nearest hundredth.
(a) 3.14738 goes to 3.15 (b) 0.73372 goes to 0.73 (c) 0.0032 goes to 0.00

Example 2. Round to the nearest ten.
(a) 817 goes to 820 (b) –1123 goes to –1120 (c) 74.567 goes to 70
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Example 3. **Important idea: Sequential rounding does not always give correct results.** You must do all the necessary rounding in one step in order to obtain correct results.

Problem: Round 64.7 to the nearest ten.

Correct Solution: The nearest ten means that the answer must either be 60 or 70. Since 4.7 is less than half of ten, then when we round to the nearest ten, we have 60.

Incorrect solution: If we round 64.7 to the nearest one first, it is between 64 and 65, and 0.7 is more than half, so it rounds to 65. Then if we round that result to the nearest ten, it is between 60 and 70 and 5 is half of ten, so we round this to 70.

Notice that we did not obtain the same answer by these two solution methods. Yet, clearly, 64.7 is closer to 60 than to 70, so the first solution method must be correct. This illustrates the incorrect answer that sometimes arises if we do our rounding in sequential steps rather than all in one step.

Discussion. What about one-half?

Bookkeepers have noticed that, if you systematically round all numbers to the nearest dollar, rounding half-dollars up, and then take sums of those rounded numbers to estimate the sums of the original values, those estimates are a bit too high to be accurate. The problem is that there is some non-symmetry in the rounding rule. All those less than half "go down" and all those more than half "go up." So far that's symmetric. The problem is that the ones that are exactly half all go the same direction, which is "up." So that's not symmetric. So the rounded values are, on the average, overall, just a little bit higher than the original numbers.

Going deeper. In situations where dealing with one-half in a non-symmetric manner might be a problem, a more sophisticated rounding rule is adopted. See the course web pages for additional discussion of more sophisticated rounding rules.

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Section 3. Precision of a number

In elementary school we learned that doing arithmetic with the number 18 and the number 18.0 gives the same results. But when we are thinking of approximate numbers, those two ways of reporting a number do not imply the same thing. In particular, they imply a different rounding precision for the number, so they imply a different set of actual values that could have led to this rounded number. These are important distinctions when we work with measured numbers.

Review. Names of the places in a number.

Example 1. Consider the number 38,145. The places, from the right, are the ones, tens, hundreds, thousands, ten thousands. So this number is

5 ones
4 tens
1 hundred
8 thousands
+ 3 ten-thousands
38,145
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Example 2. Consider the number 1.2479. The left-most place, before the decimal point, is the ones place. Immediately after the decimal is the tenths place, then the hundredths place, then the thousandths place, then the ten-thousandths place.

1 one
2 tenths
4 hundredths
7 thousandths
\+ 9 ten-thousandths
1.2479

Precision: We can define the precision of a number in three different ways.

1. State it in words.
2. State it with a number.
3. Imply it by how the number is written.

Example 3: The amount $5200 is measured to the nearest hundred dollars. That can be stated as

1. The amount $5200 is measured to the nearest hundred dollars.
2. The amount $5200 is measured to the nearest 100 dollars.
3. The value $5200 has the obvious implied precision.

Example 4: The number 73.123 is rounded to the nearest one-thousandth. That can be stated as

1. The number 73.123 is rounded to the nearest one-thousandth.
2. The number 73.123 is rounded to the nearest 0.001.
3. The number 73.123 has the obvious implied precision.

Example 5: Round 18.038 to the nearest tenth.
Solution: Since we need to cut off everything past the tenths place, we must cut off the marked-out part here 18.0~~38~~. Since the 3 at the beginning of the 38 is less than 5, we round down and the answer is 18.0. We could communicate that as "The answer is 18, rounded to the nearest tenth." But no one would do that because it is confusing. It is much less confusing if we always include the tenths digit when we report the result of rounding to the nearest tenth. So here we could say "the answer is 18.0, rounded to the nearest tenth." In fact, technical people would not give the words here because everyone would understand that the number 18.0 implies that it is rounded to the nearest tenth.

Example 6. Round 2.1397354 to the nearest 0.001.
Solution. Since we need to cut off everything past the thousandths place, we cut off the marked-out part here 2.139~~7354~~ and notice that the first digit of the 7354 that we need to cut off is greater than half, so we must round up. But the previous digit is 9, so to round up we must go to 10. That means the answer is 2.140 since the 3 in front of the 9 must go up 1. The answer is 2.140.

Example 7: We measured the length of pipe as 6.30 meters. What is the implied precision of that number?
Solution: Since this number is given in hundredths of a meter, and there is no reason to write the zero on the end except to give the hundredths place, this implies that the number is measured to the nearest hundredth, that is, the nearest 0.01.

Example 8: In this summary of a financial report of a small business, the amount spent on utilities last year is given as $13,000. What is the implied rounding precision of that number?
Solution: The implied rounding precision is that the number is rounded to the nearest thousand dollars.

Caution: (Extending the previous example.) If the actual value was $12,973.37 and we wanted to round to the nearest hundred dollars, the answer would also be $13,000. In our previous examples, where we were doing all the rounding somewhere after the decimal point, we could easily communicate the rounding precision by extra zeros as needed, so the rounding precision could always be given unambiguously. But **when we have large numbers, where the extra zeros are not after the decimal, then the rounding precision cannot always be given unambiguously by just writing the number**. In those cases, if we want to be completely clear in reporting a rounded number, we must state the rounding precision in words or numbers and not rely on our readers just using the implied rounding precision. When we are reading rounded numbers reported by others, we should look at other information given besides the actual number to see whether the implied rounding precision is consistent with that.

When scientists and others working in technical fields report approximate numbers, they sometimes use the concept of identifying significant digits. **The significant digits in a number are those which give an actual value that was measured or recorded, as opposed to the digits in a number which merely to indicate the size of the number**. (Think of the usual meaning of the word "significant." These digits are significant because they are actually measured.) Identifying the significant digits is simply another way of identifying the implied precision of the number.

Number	**Implied precision in words**	**Implied precision in numbers**	**Significant digits underlined**
17.3	Tenths	0.1	17.3
18	Ones	1	18
18.0	Tenths	0.1	18.0
100.6	Tenths	0.1	100.6
83.20	hundredths	0.01	83.20
97.1080	Ten-thousandths	0.001	97.1080
13000	Thousands	1000	13000
20800	Hundreds	100	20800

In identifying significant digits, we have focused here mainly on identifying the right-most significant digit to be consistent with the implied rounding precision. However, when we change the units of a measurement after the measurement has been made, we don't want that to change the number of significant digits. Specifically notice the second number in our previous table.

Measurement and its precision	**Completely clear correct report in centimeters**	**Fairly clear correct report**	**Not a clear report**	**Completely clear correct report in meters**	**# of sig digits**
12 cm, 1	12 cm			0.12 m	2
2.7 cm, 0.1	2.7 cm			0.027	2
18.2 cm, 0.1	18.2 cm			0.182 m	3
33 cm, 0.01	33.0 cm			0.330 m	3
30 cm, 1		30. cm		0.30 m	2
Estimate of 20 cm, 10		20 cm		0.2 m	1
Estimate of 180 cm, 10		180 cm		1.8 m	2
Estimate of 300 cm, 10			300 cm	3.0 m	2
Estimate of 300 cm, 100		300 cm		3 m	1

Some books give "rules for identifying significant digits." These include

1. leading zeros in a decimal number less than 1 are not significant
2. trailing zeros in a number greater than 1 are not significant

The reason behind these rules is that, unless you are specifically told otherwise, these zeros are in the number just to indicate the size of the number and not to indicate an actual measured value. The second and last rows in the table are examples of these situations.

Going Deeper:

In scientific and technical work, the measurement units will usually be chosen so that the rounding precision is either to the nearest whole number or is in the decimal portion so that it can be given unambiguously. Sometimes they will use scientific notation to convey the rounding precision unambiguously. See the course web pages for supplemental information for this Topic. This includes a discussion of how scientists and other technical workers usually choose measurement units so that the implied precision is almost always at the level of 1, 0.1, or 0.01, so that the numbers' implied precision is easy to read and unambiguous. Some discussion of the use of scientific notation is also included.

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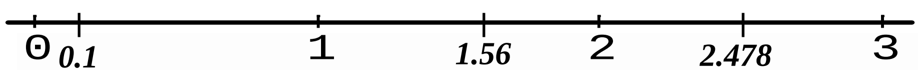
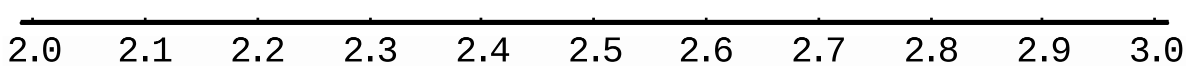
Section 4. Write an interval of actual values that are consistent with a rounded value.

We can represent all possible real numbers on a number line. We can only label a few of the points because if we tried to label more, the picture would become confusing. But you should think of being able to graph on a number line any particular number you are given. (When doing these problems, estimate distances rather than trying to measure them exactly.)

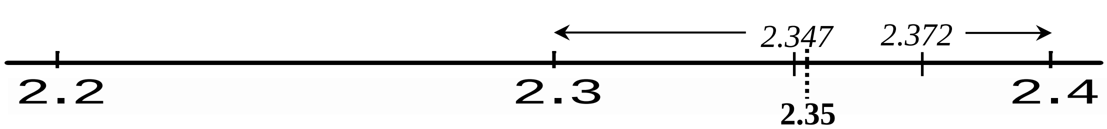
Example 1. Draw a number line that includes the values from 0 to 3. On that number line, label the points corresponding to the three numbers 0.1, 1.56, and 2.478.



Example 2. Draw a number line that includes the values from 2.0 to 3.0 and label the points corresponding to these numbers: 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, and 3.0.



Example 3. Draw a number line that includes the values from 2.2 to 2.4, and label 2.2, 2.3, and 2.4. (We will now take some numbers in that range with more decimal places, graph them on the number line, and then consider how to round each of those numbers to the nearest tenth.)
Consider the two numbers 2.347 and 2.372. Place each of them reasonably correctly on the number line. Round each of these numbers to the nearest tenth and use an arrow to show which number it is rounded to. What is the cut-off value that separates the numbers that are rounded to 2.3 and those that are rounded to 2.4? Indicate that value on the number line and label it.



Notice that the cut-off value separating the numbers that are rounded to 2.3 and those rounded to 2.4 is 2.35, which is halfway between 2.3 and 2.4.

Example 4. We will draw the interval of actual values that are consistent with the rounded value of 2.3.
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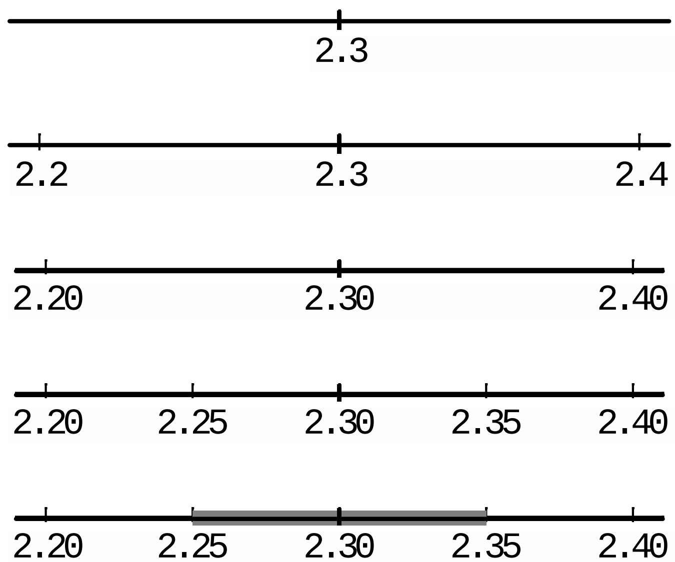
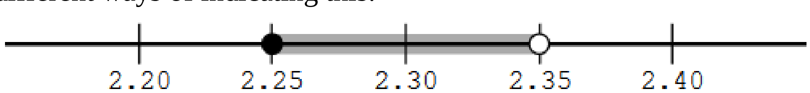
Procedure:

- Think of numbers near 2.3 that have the same precision. Since the precision of 2.3 is one-tenth, then other nearby numbers are 2.1, 2.2, 2.3, 2.4, 2.5, etc.
- Choose numbers with the same precision – the given 2.3 and the ones on either side, which are 2.2 and 2.4. Put them on a number line.
- What is the cut-off value that separates the number that are rounded to 2.2 and the numbers that are rounded to 2.3? Indicate that value on the number line.
- To label that value, it is easiest to go back and put an extra zero on all the other values, so you have 2.20, 2.30, and 2.40. That helps clarify that the value half-way between 2.20 and 2.30 is 2.25. Label it.
- What is the cut-off value that separates the numbers that are rounded to 2.3 and the numbers that are rounded to 2.4? Indicate that value on the number line and label it.
- Indicate the interval of values that are rounded to 2.3.



Discussion: What about the end points of this interval?

By drawing these intervals and end points, we see that numbers between 2.25 and 2.35, when rounded to the nearest tenth, are rounded to 2.3.
But, of course, we remember that the exact value 2.35, when rounded to the nearest tenth, goes to 2.4. So we really shouldn't include 2.35 in the interval.
So how far back should we go? Well, what about rounding 2.349999 to the nearest tenth? That goes to 2.3.
So we need all numbers less than 2.35 to be rounded to 2.3. We need some illustration or notation to indicate all the numbers up to, but not including 2.35.
These are several different ways of indicating this:



Interval [2.25,2.35)

2.25 <= actual < 2.35

Each of these indicates that the left endpoint is included and the right endpoint is not included.

In this course, when we discuss the interval of values of the actual data that are consistent with a rounded value, it will be adequate for you to merely give the endpoints and not specifically indicate that the left end point is included and the right end point is not.

Example 5. Consider numbers rounded to the nearest hundredth.

a. Write several numbers between 7.6000 and 7.6500 and round each of them to the nearest hundredth.
b. Draw a number line and use it to illustrate which numbers are to be rounded to 7.63.
c. Suppose we report a number as 7.63, rounded to the nearest hundredth. Write an interval that gives the actual values that are consistent with a rounded value of 7.63

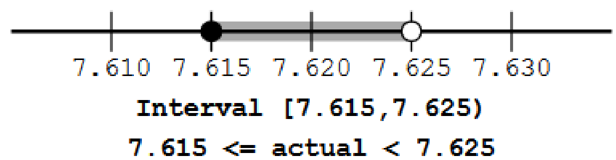
Solution:

a.

7.6278 goes to 7.63
7.6421 goes to 7.64
7.6189 goes to 7.62
7.6317 goes to 7.63
7.6048 goes to 7.60

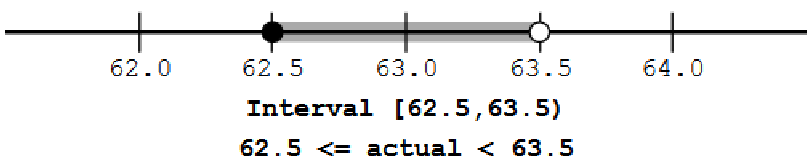
7.6251 goes to 7.63
7.6250 goes to 7.63
7.6249 goes to 7.62

b. and c.



Example 6: If a measured number is reported as 63 feet, rounded to the nearest foot, what interval of possible actual values are consistent with that?

Solution: (All numbers here are in feet.)



Method: To determine what range of actual values are consistent with a given rounded number,

a. Write the given rounded number and several numbers near it with the same precision as the given number.
b. Graph a number line with the next-smaller and the next-larger number with the same precision on either side of the given rounded number.
c. If the numbers end in decimals, then add a zero at the end of each number in order to make it easier to label the numbers halfway in between.
d. On that number line, cut the intervals between the two rounded numbers in half.
e. Label those two cut-off points.
f. Mark the interval between those two cut-off points as the range of actual values that are consistent with the rounded number.
g. To be completely accurate in writing the interval, indicate that the smaller of the two cut-off values is included in the interval and the larger of the two cut-off values is not included in the interval.

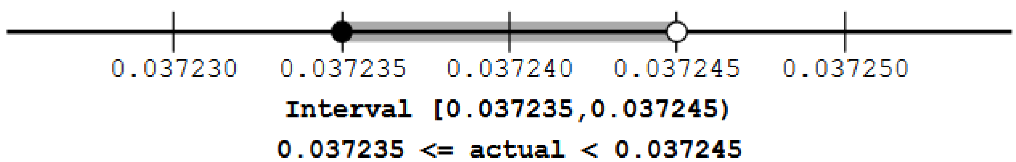
Precision of end points: The numbers at the ends of the interval of actual values will **always** have exactly one more decimal place of precision than the given rounded number.

Discussion: Notation.

In the examples below, the same answer is given with several different forms of notation. You do not have to write all of those different ways for every answer. You must understand all the ways, but pick the one you prefer and write your answers with just that one form of notation.

Example 7: A measured number is reported as 0.03724 kilometers
a. state the implied precision as a number
b. state the implied precision in words
c. underline the significant digits
d. find the interval of possible actual values consistent with this rounded number.

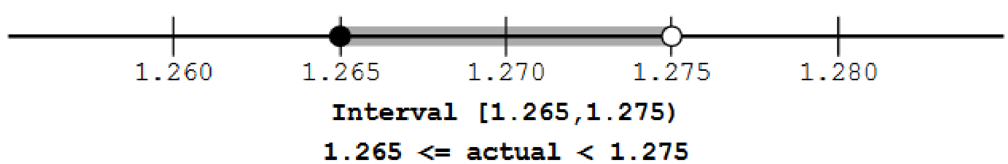
Solution:
a. The precision here is implied to be 0.00001
b. The precision is one hundred-thousandth.
c. The significant digits are underlined: 0.0$\underline{3724}$ kilometers
d. The next-smaller number with the same precision is 0.03723 and the next-larger number with the same precision is 0.03725. Write those as 0.037230 and 0.037240 and 0.037250. Then find and label the half-way points.



The actual number is between 0.037235 and 0.037245 kilometers.

Example 8: A measured number is reported as 1.27 liters.
a. state the implied precision as a number
b. state the implied precision in words
c. underline the significant digits
d. what interval of possible actual values are consistent with that and what are several ways this might be reported?

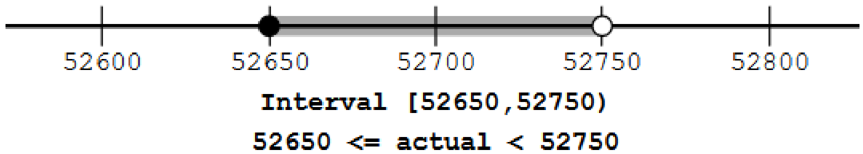
Solution:
a. The rounding precision here is implied to be 0.01.
b. The precision is one-hundredth.
c. The significant digits are underlined: $\underline{1.27}$ liters.
d. The next-smaller number with the same precision is 1.26 and the next-larger number with the same precision is 1.28. Write those as 1.260, 1.270, and 1.280. Then find and label the half-way points.



Or it might be reported as 1.27 ± 0.005. liters

Another method of reporting it would be $1.27^{-0.005}_{+0.005}$ liters, which is usually only used if the distances on the two sides are not equal.

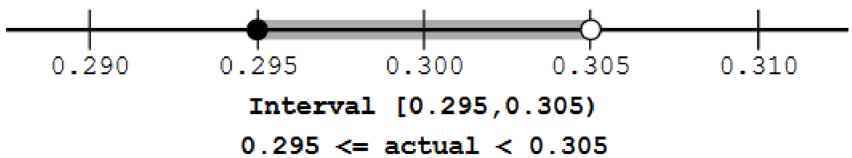
Example 9: If a measured number is reported as 52700 feet, rounded to the nearest hundred feet, underline the significant digits and identify the interval of possible actual values.
Solution: (All numbers here are in feet.) The significant digits are underlined: $\underline{527}$00 feet



This might also be reported as 52700 ± 50 feet

Another method of reporting it would be 52700^{-50}_{+50} feet, which is usually only used if the distances on the two sides are not equal.

Example 10: If a measured number is reported as 0.30 meters, what is the rounding precision, underline the significant digits, and find the interval of possible actual values.
Solution: . The rounding precision is 0.01, which is one-hundredth. The significant digits are underlined: 0.30 The next-smaller rounded number is 0.29 and the next-larger rounded number is 0.31. We'll write those as 0.290 and 0.300 and 0.310. (All numbers here are in meters.)



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This might also be reported as 0.30 ± 0.005 meters

Another method of reporting it would be $0.30^{-0.005}_{+0.005}$ meters, which is usually only used if the distances on the two sides are not equal.

Example 11: If a measured number is reported as 12 meters,
a. what is the interval of actual values consistent with that,
b. what is the maximum amount that the actual value could differ from the reported value
c. what is the maximum amount of error as a percentage of the reported number?
Solution:
a. The interval of actual values is between 11.5 and 12.5 meters.
b. The maximum amount that the actual value could differ from the reported value is 0.5 meters.
c. To find the percentage of the maximum error as a percentage of the reported value, we must write it as a fraction, and then do the division and get a decimal fraction. Then convert that decimal to a percentage. $\frac{0.5 \text{ m}}{12 \text{ m}} = 0.041666667$, which is 4.16666667%.
So **the maximum error is about 4% of the reported value**. This is called the *relative error*.

Example 12: If a measured number is reported as 12.0 meters,
a. what is the interval of actual values consistent with that,
b. what is the maximum amount that the actual value could differ from the reported value
c. what is the maximum amount of error as a percentage of the reported number?
Solution:
a. The interval of actual values is between 11.95 and 12.05 meters.
b. The maximum amount that the actual value could differ from the reported value is 0.05 meters.

c. To find the percentage of the maximum error as a percentage of the reported value, we must write it as a fraction, and then do the division and get a decimal fraction. Then convert that decimal to a percentage. $\frac{0.05 \text{ m}}{12 \text{ m}} = 0.0041666667$, which is 0.416666667%.

So **the maximum error is about 0.4% of the reported value**. This relative error is one-tenth of the relative error in Example 11, reflecting the increased precision of the reported number.

Exercises:
In the examples 7-12 above, the same answer was given with several different forms of notation. You do not have to write all of those different ways for every answer, unless you are explicitly asked to give several different forms of the answer. You must understand how to write all the forms, of course.

Part I.

1. Round to the nearest hundredth. (a) 3.14738 (b) 0.73372 (c) 0.0032

2. Round to the nearest ten. (a) 817 (b) –1123 (c) 74.567

3. Round 64.7 to the nearest ten.
   a. Do it in one step, correctly.
   b. Illustrate the error that occurs if you try to do it in two steps – first rounding to the nearest one and then rounding that result to the nearest ten.

4. For each of the following numbers, fill in the blanks in the table.

Number	**Implied precision in words**	**Implied precision in numbers**	**Significant digits underlined**
17.3			
18			
18.0			
100.6			
83.20			
97.1080			
13000			
20800			

5. Draw a number line that includes the values from 0 to 3. On that number line, label the points corresponding to the three numbers 0.1, 1.56, and 2.478.

6. Draw a number line that includes the values from 2.0 to 3.0 and label the points corresponding to these numbers: 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, and 3.0.

7. Draw a number line that includes the values from 2.2 to 2.4, and label 2.2, 2.3, and 2.4. Then find the cut-off values for the numbers that would round to 2.3.

8. Consider numbers rounded to the nearest hundredth.
   a. Write several numbers between 7.6000 and 7.6500 and round each of them to the nearest hundredth.
   b. Draw a number line and use it to illustrate which numbers are to be rounded to 7.63.
   c. Suppose we report a number as 7.63, rounded to the nearest hundredth. Write an interval that gives the actual values that are consistent with a rounded value of 7.63

9. If a measured number is reported as 63 feet, rounded to the nearest foot, what interval of possible actual values are consistent with that?

10. A measured number is reported as 0.03724 kilometers
    a. state the implied precision as a number
    b. state the implied precision in words
    c. underline the significant digits
    d. find the interval of possible actual values consistent with this number.

11. A measured number is reported as 1.27 liters
    a. state the implied precision as a number
    b. state the implied precision in words
    c. underline the significant digits
    d. find the interval of possible actual values consistent with this number and what are several ways this might be reported?

12. Pay careful attention to when trailing zeros are conveying precision.
    a. If a measured number is reported as 52700 feet, rounded to the nearest hundred feet, underline the significant digits and find the interval of possible actual values that are consistent with that.
    b. If a measured number is reported as 0.30 meters, rounded to the nearest hundredth of a meter, underline the significant digits and find the interval of possible actual values that are consistent with that.

13. If a measured number is reported as 12 meters,
    a. what is the interval of actual values consistent with that,
    b. what is the maximum amount that the actual value could differ from the reported value
    c. what is the maximum amount of error as a percentage of the reported number?

14. If a measured number is reported as 12.0 meters,
    a. what is the interval of actual values consistent with that,
    b. what is the maximum amount that the actual value could differ from the reported value
    c. what is the maximum amount of error as a percentage of the reported number?

Part II.

15. Consider 12.1464.
    a. Round 12.1464 to the nearest hundredth.
    b. Round 12.1464 to the nearest tenth.
    c. Round 12.1464 to the nearest hundredth and then round that result to the nearest tenth. Is this a good method of getting a rounded number for 12.1464 to the nearest tenth? Explain.
16. Consider 8.5478
    a. Round 8.5478 to the nearest hundredth.
    b. Round 8.5478 to the nearest tenth.
    c. Round 8.5478 to the nearest hundredth and then round that result to the nearest tenth. Is this a good method of getting a rounded number for 8.5478 to the nearest tenth? Explain.

For each of the given numbers in problems 17-24,

- d. state the implied rounding precision as a number
- e. state the implied rounding precision in words
- f. underline the significant digits
- g. find the interval of possible actual values consistent with this rounded number.
- h. what is the maximum amount that the actual value could differ from the reported value?
- i. what is the maximum amount of error as a percentage of the reported number?

17. The rounded number 0.71
18. The rounded number 8.93
19. The measured number $54,000
20. The measured number $157,000
21. The measured number 2.347 liters
22. The measured number 18.978
23. The measured number 0.0072
24. The measured number 0.0378

For each of the given numbers in problems 25-28,

- a. underline the significant digits
- b. find the interval of possible actual values consistent with this rounded number.

25. The measured number 0.00406
26. The measured number 0.0003802
27. The measured number 12.000406
28. The measured number 9.0382

29. Consider these three rounded numbers
    - a. For the measured number 7 meters, underline the significant digits and find the interval of actual values.
    - b. For the measured number 7.0 meters, underline the significant digits and find the interval of actual values.
    - c. For the measured number 7.00 meters, underline the significant digits and find the interval of actual values.
    - d. Do these three reported rounded numbers convey identical information?
30. Consider these three rounded numbers.
    - a. For the measured number 82 meters, underline the significant digits and find the interval of actual values.
    - b. For the measured number 82.0 meters, underline the significant digits and find the interval of actual values.
    - c. For the measured number 82.00 meters, underline the significant digits and find the interval of actual values.
    - d. Do these three reported rounded numbers convey identical information?