Spreadsheets in a Math for Liberal Arts Course

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Challenge and Opportunity

Challenge

It’s hard to put much math modeling into an already-full math course.

Opportunity

Math for Liberal Arts has more flexibility in the choice of topics and depth of coverage than other lower-division courses.

Challenge and Opportunity

Challenge

Students in Math for Liberal Arts are, in general, not adept at algebra.

Opportunity

It is common that students feel more powerful using spreadsheets to investigate quantitative questions than they have felt in the past using algebra.

Types of spreadsheets

Most of the course:

- Rule-driven procedures.
  Make the spreadsheet just by implementing the rule and iterating.

Latter part of the course:

- Graphing formulas.
- Using a graph to estimate the best fitting formula to data.
  This gives students more skill in interpreting and understanding the parameters of the formulas.

How do we build skills?

1. Start with some basic review of percentages and proportions. Using a "Problem solving" chapter from a text at this level, quickly go to more complicated problems involving these.
2. Teacher models using a spreadsheet for calculation on problems that students are using calculators for.
3. Teacher starts demonstrating cell references and pulling down formulas.
4. Students work on problems that are straightforward to do one step at a time, but tedious to fully carry out by hand.

Days 1-3

Group work in “Problem Solving” from an open-source text by David Lippman *Math in Society*. (percents, ratios and proportions, unit conversions, estimation)

About 40% straightforward problems:

- Ireland has a 23% VAT (value-added tax, similar to a sales tax). How much will the VAT be on a purchase of a €250 item?

About 60% more sophisticated problems:

- Another project on Kickstarter for an iPad stylus raised 1,253% of their goal, raising a total of $313,490 from 7,511 supporters. What was their original goal?
- Which is better: having a stock that goes up 30% on Monday than drops 30% on Tuesday, or a stock that drops 30% on Monday and goes up 30% on Tuesday? In each case, what is the net percent gain or loss?

Spreadsheets

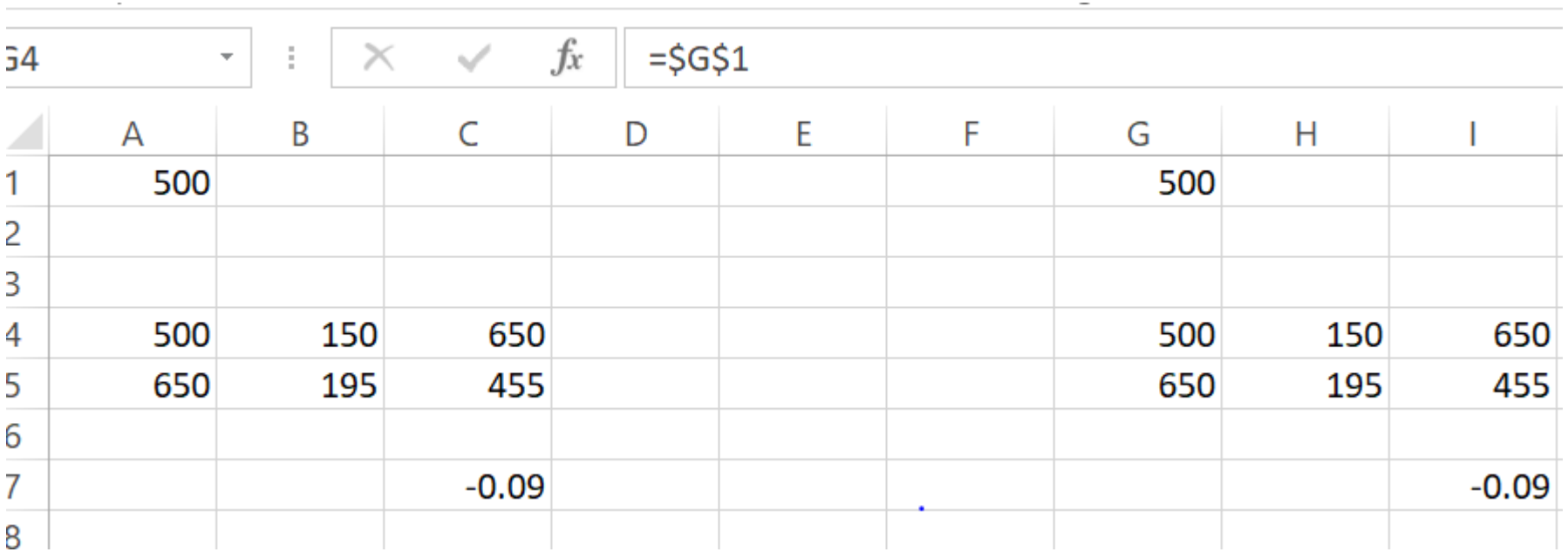
In days 1-3, most students use a calculator.

I show the solutions to some problem after they have worked on them.
I show the calculations using a spreadsheet, with little description.

(“Use = at the beginning of the cell to do the computation.”
“I like to think of a spreadsheet as my “smart scratch paper.”)

For the last problem on the previous page, I illustrate using a cell-reference in the formula so that I can easily compute it for several different initial values. The students usually seem impressed by that.

For the problem about increase and decrease by 30%, I illustrate using a cell-reference in the formula so that I can easily compute it for several different initial values. The students usually seem impressed by that.



	A	B	C	D	E	F	G	H	I
1	500						500		
2									
3									
4	500	150	650				500	150	650
5	650	195	455				650	195	455
6									
7			-0.09						-0.09
8									

Day 4

Question: Suppose in 2010, there were two different jobs available and you could have predicted, very accurately, what the increase in pay for each of those jobs would be for the next 11 years. (To make this easier to handle numerically, call the year 2010 year 0, and then 2011 is year 1, etc.)
Which job would be the better one for you to be in for that eleven-year period?

- Job A. Starting salary is $26000 per year and the increase will be 7.2% per year.
- Job B. Starting salary is $30000 per year and the increase will be $1000 per year.

Day 4 Instructions:

1. How will you measure which job is better, in terms of the pay? Be specific.
2. How will you calculate what is needed to figure that out?
3. Calculate it and answer the question.
4. Make a graph to illustrate how the two salaries compare for each of the years.
5. What does the graph tell you about when the salaries are closest together?
6. Suppose you have now answered that question. Here's another question: How much would the starting salary at the other job have to have increased in order to make the decision go the other way?

Days 5-6

Other problems of the same type.

- Given a set of quiz grades and test grades, and a grading scheme, figure out what grade is needed on the final exam to earn a B. And then, a new grading scheme where the lowest three quiz grades are dropped, and the same question. (Here we learn to sort a column.)

- What level of annual spending at Costco makes it a good deal to buy the “Executive” membership?

“Cut-off” problems

Students use instructor-prepared spreadsheets to investigate

- Finding the optimal cut-off credit score for granting a loan.
  (Balance cost of forfeited loans with revenue from loans which were paid off.
  Input is expected frequency of, and forfeiture rate for, the various credit scores.)

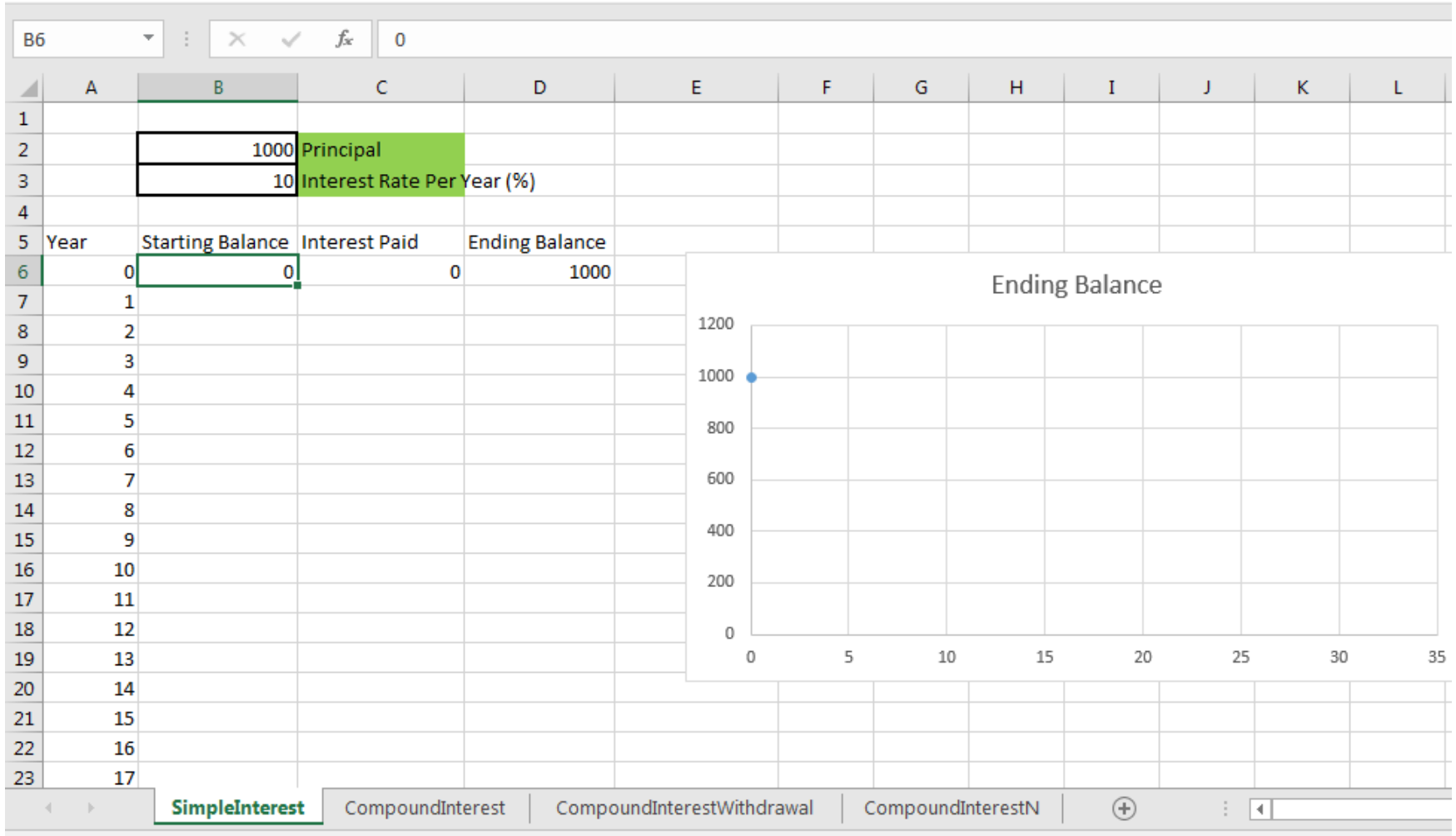
- Finding the optimal number of overbooked seats on an airline.

What do students learn to do with spreadsheets in the Finance portion?

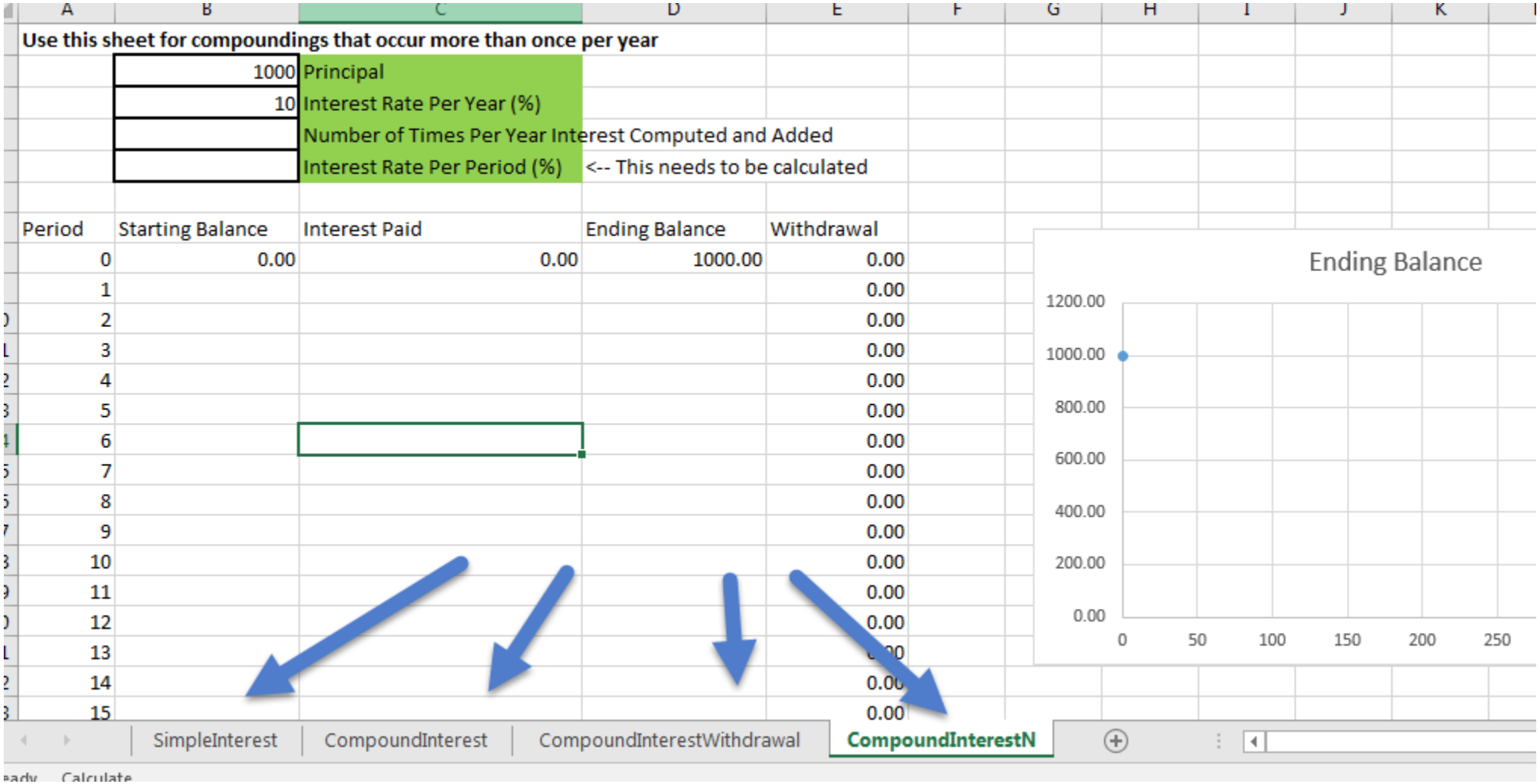
- Develop spreadsheets to compute accumulated savings with simple and compound interest.

- **Method:**
  By hand, compute the values one row at a time, for two or three rows.
  Then write formulas in the spreadsheet.

We give the framework, students enter formulas and pull down.



Students model various types of growth.



Compound Interest Spreadsheets

- Problems solved with these sheets include
- If the initial amount is $4000, at 3.5% per year, compounded monthly, how much will you have in the account at the end of 15 years?
- If the initial amount is $4000, at 3.5% per year, compounded monthly, and, at the end of the 6^{th} year, you withdraw $2500, is the amount you have at the end of 15 years lowered by exactly $2500 or more or less than that? Revise your spreadsheet to enable you to answer questions like this easily.

Continuing Finance

In class, students also set up a loan amortization table themselves, including a column for extra payments. (Starting on paper, then formulas in a spreadsheet.)

Some useful spreadsheets are not reasonable for students to set up in this "rule-driven" computation method. I give them spreadsheets for these.

- Loan payment calculator
- Savings annuity calculator
- Payout annuity calculator

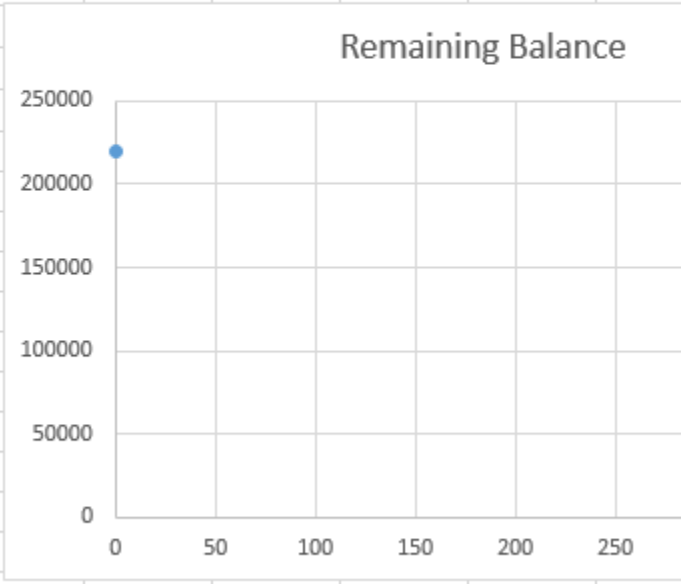
They have quite a lot of homework in which they choose the appropriate spreadsheet calculator and answer the questions.

Students develop a loan amortization sheet.

Create an amortization with the option for extra payments toward the principal

220000	Principal (Loan Amount)
4	Interest Rate (%)
30	Number of Years Over Which Loan Repaid
12	Number of Payments Per Year
1050.31	Periodic Payment

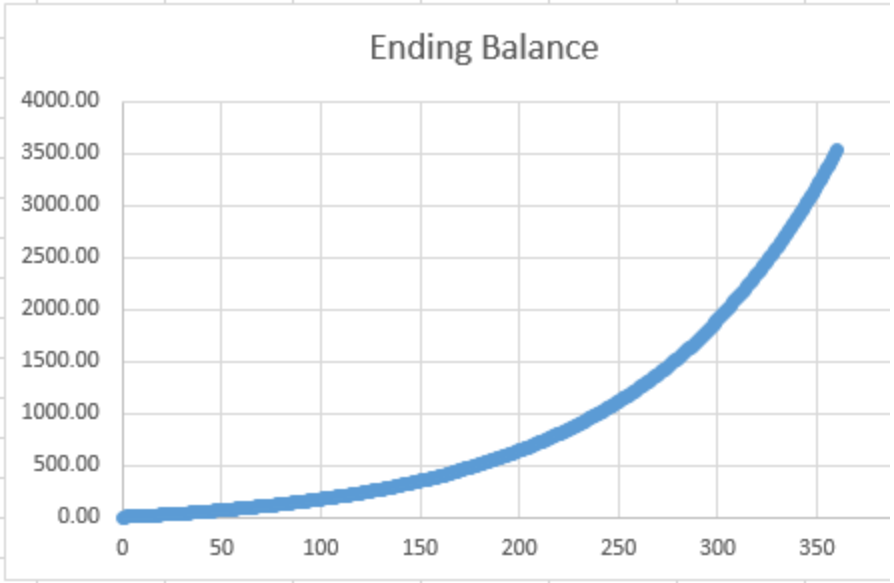
Period	Starting Balance	Interest Paid	Principal Paid	Remaining Balance	Extra Payment	Accumulated Interest
0	220000	0.00	0.00	220000	0.00	
1					0.00	
2					0.00	
3					0.00	
4					0.00	
5					0.00	
6					0.00	
7					0.00	
8					0.00	
9					0.00	
10					0.00	
11					0.00	



Stack Plot: Breakdown of Monthy Pa

Students use an annuity calculator to explore the effects of various interest rates and various choices.

	A	B	C	D	E	F
1	**Use this calculator for fixed deposits.**					
2		0	St… g Balance			
3		1	Pc… e Deposit			
4		1	N… er of Deposits Per Year			
5		1	A… as a %)			
6						
7	Deposit Number	Starting Balance	Deposit	Interest	Ending Balance	Withdrawal
8	0	0.00	0.00	0.00	0.00	0.00
9	1	0.00	1.00	0.01	1.01	0.00
10	2	1.01	1.00	0.02	2.03	0.00
11	3	2.03	1.00	0.03	3.06	0.00
12	4	3.06	1.00	0.04	4.10	0.00
13	5	4.10	1.00	0.05	5.15	0.00
14	6	5.15	1.00	0.06	6.21	0.00
15	7	6.21	1.00	0.07	7.29	0.00
16	8	7.29	1.00	0.08	8.37	0.00
17	9	8.37	1.00	0.09	9.46	0.00
18	10	9.46	1.00	0.10	10.57	0.00
19	11	10.57	1.00	0.12	11.68	0.00
20	12	11.68	1.00	0.13	12.81	0.00
21	13	12.81	1.00	0.14	13.95	0.00
22	14	13.95	1.00	0.15	15.10	0.00
23	15	15.10	1.00	0.16	16.26	0.00

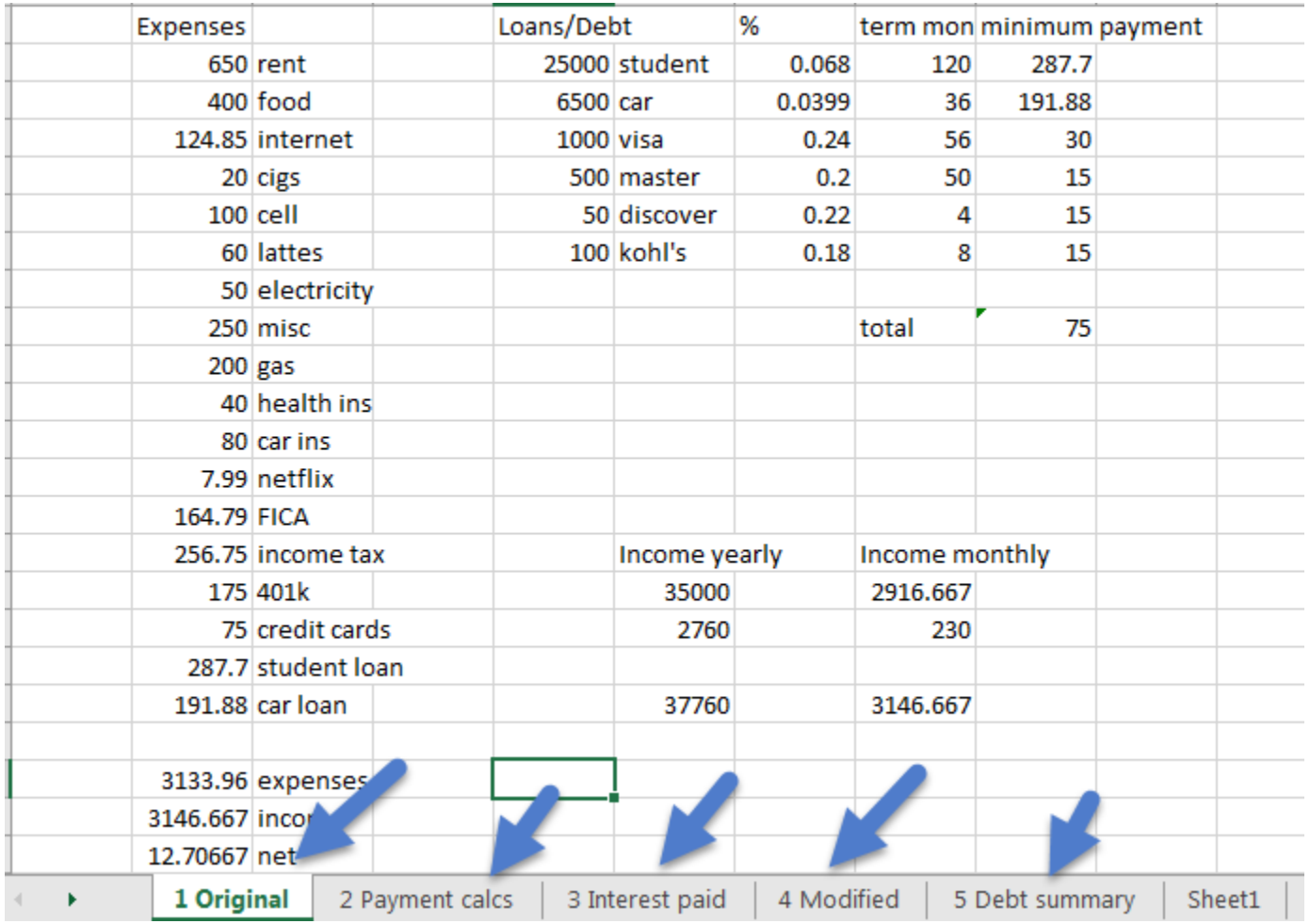


AnnuityCalculatorFixedDeposits | AnnuityCalcVariableDeposits

Financial Advice Project (in Groups)

- Students are given Joe's income, monthly spending pattern, and debts and asked to give him advice. (Three different scenarios per group.)
- They are expected to use an online debt-reduction calculator to compute his optimal loan payment strategy, including choosing which method to use to decide how much to pay on each loan and when (snowball, avalanche, etc.)
- They are also expected to use a spreadsheet to summarize all of this.
- On the final exam in the course, they are expected to use their spreadsheet to make some additional adjustments and give the resulting values.

One student's workbook



Expenses		Loans/Debt		%	term mon	minimum payment
650	rent	25000	student	0.068	120	287.7
400	food	6500	car	0.0399	36	191.88
124.85	internet	1000	visa	0.24	56	30
20	cigs	500	master	0.2	50	15
100	cell	50	discover	0.22	4	15
60	lattes	100	kohl's	0.18	8	15
50	electricity					
250	misc				total	75
200	gas					
40	health ins					
80	car ins					
7.99	netflix					
164.79	FICA					
256.75	income tax		Income yearly		Income monthly	
175	401k		35000		2916.667	
75	credit cards		2760		230	
287.7	student loan					
191.88	car loan		37760		3146.667	
3133.96	expenses					
3146.667	inco					
12.70667	net					

1 Original | 2 Payment calcs | 3 Interest paid | 4 Modified | 5 Debt summary | Sheet1

Her revised budget for Joe

A	B	C	D	E	F	G	H	I
	150	misc	subtracted 100 by getting rid of unneccessary misc payments					
	100	gas	subtracted 100 by using public transportation					
	40	health ins						
	80	car ins						
	164.79	FICA						
	256.75	income tax						
	175	401k						
	$ 836.90	Loans/credit cards		edited loan payments to decrease amount of interest pa				
						Income yearly		Income monthl
						35000		2916.667
						2760		230
						37760		3146.667
	$2,526.43	expenses						
	$3,146.67	income				Savings account		
	$ 620.24	net						
						Monthly deposit		Interest rate
						$ 620.24		0.01
				started a savings account with investing his monthly cash				

Modeling data with formulas

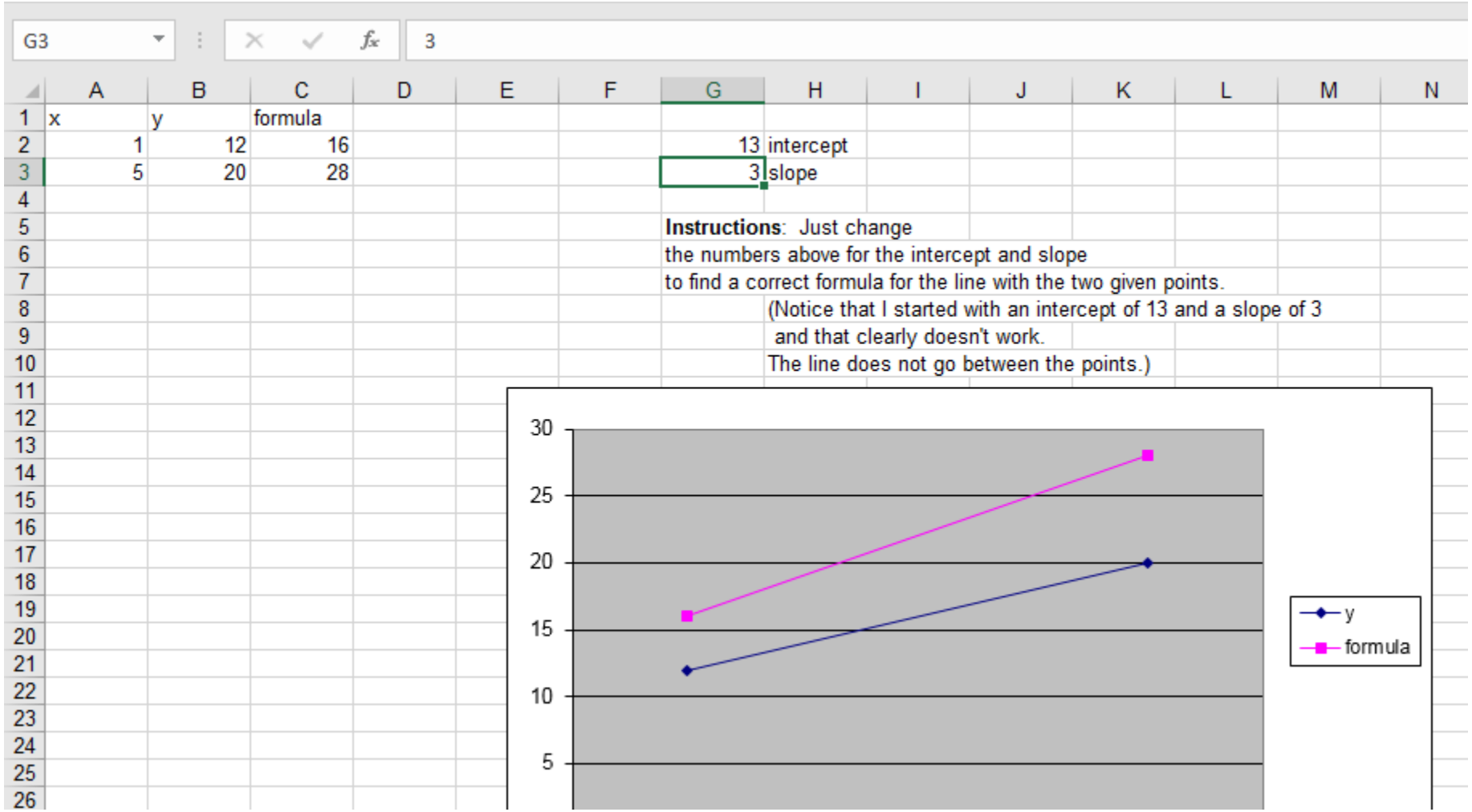
Linear $y = mx + b$

Quadratic $y = a(x - h)^2 + k$

Exponential $y = a(1 + r)^x$

These forms were chosen (for quadratic and exponential) because the parameters are easy to interpret.

Example for students to estimate slope and intercept.



G3 | 3

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	x	y	formula											
2	1	12	16				13	intercept						
3	5	20	28				3	slope						
4														
5							**Instructions**: Just change							
6							the numbers above for the intercept and slope							
7							to find a correct formula for the line with the two given points.							
8							(Notice that I started with an intercept of 13 and a slope of 3							
9							and that clearly doesn't work.							
10							The line does not go between the points.)							
11														
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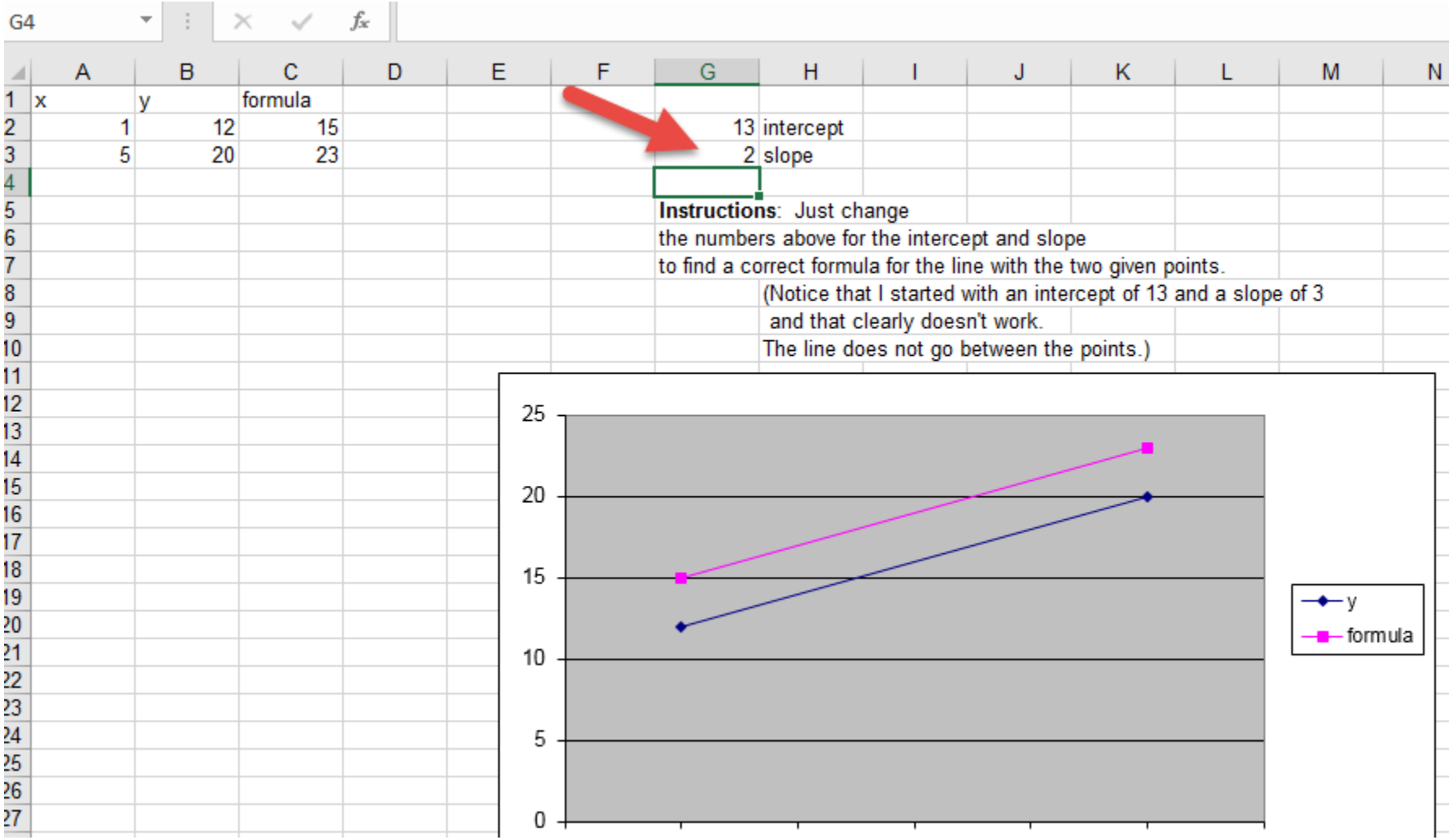
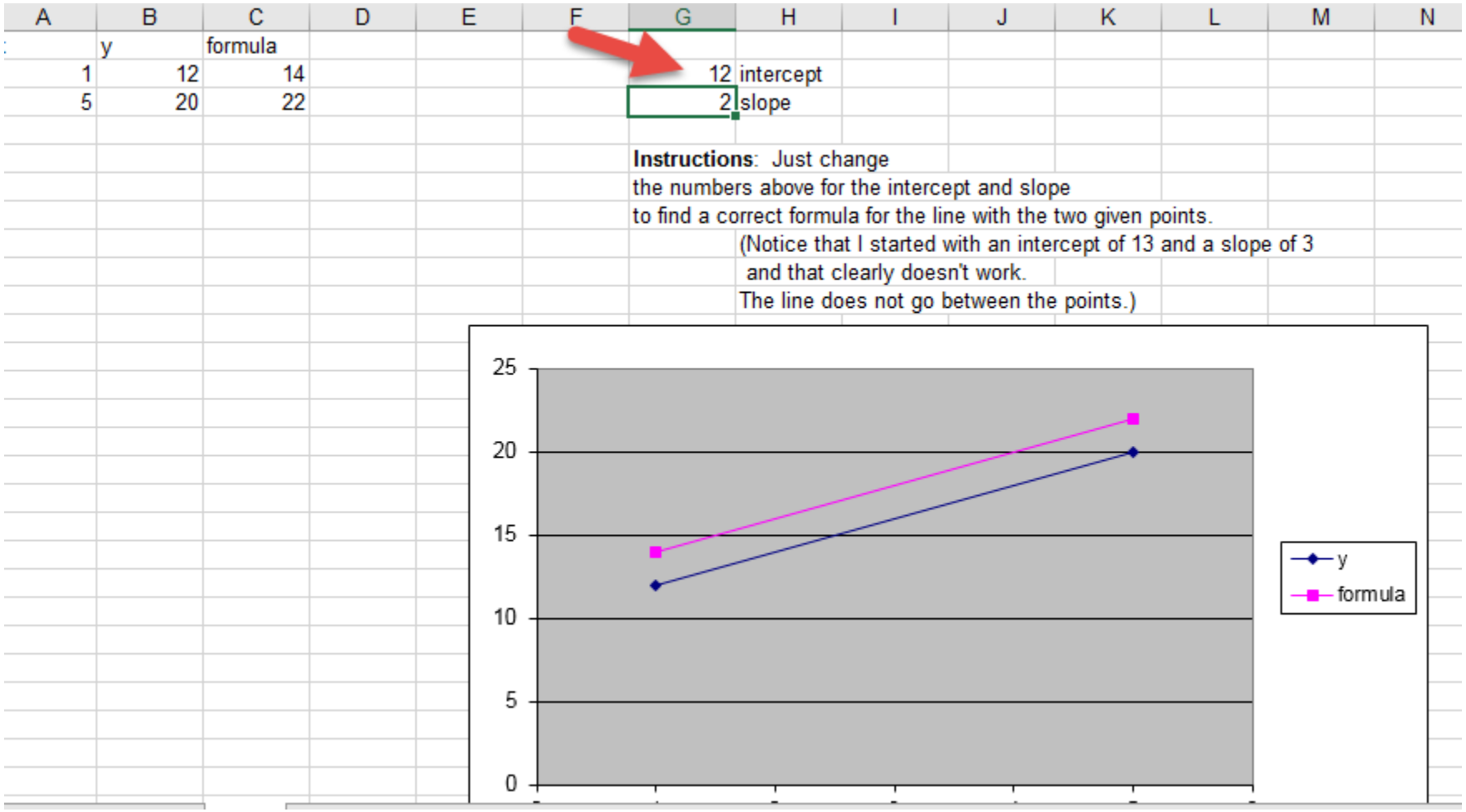
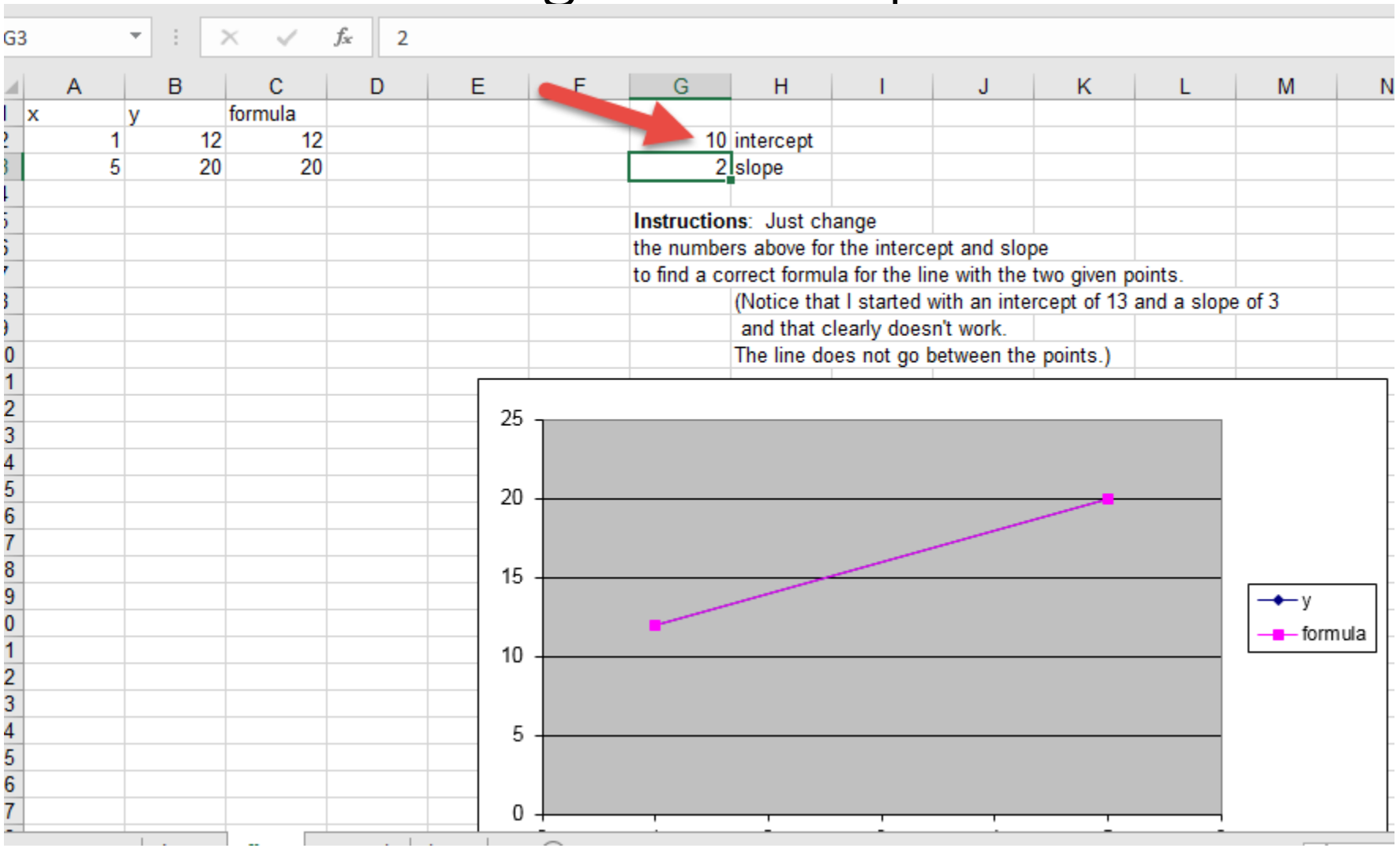
Since the slope of the formula line was too high, lower the slope to 2.



Since the intercept was too high, lower it. But I didn't lower it enough.



Kept lowering the intercept until the formula line matched the line through the two points.

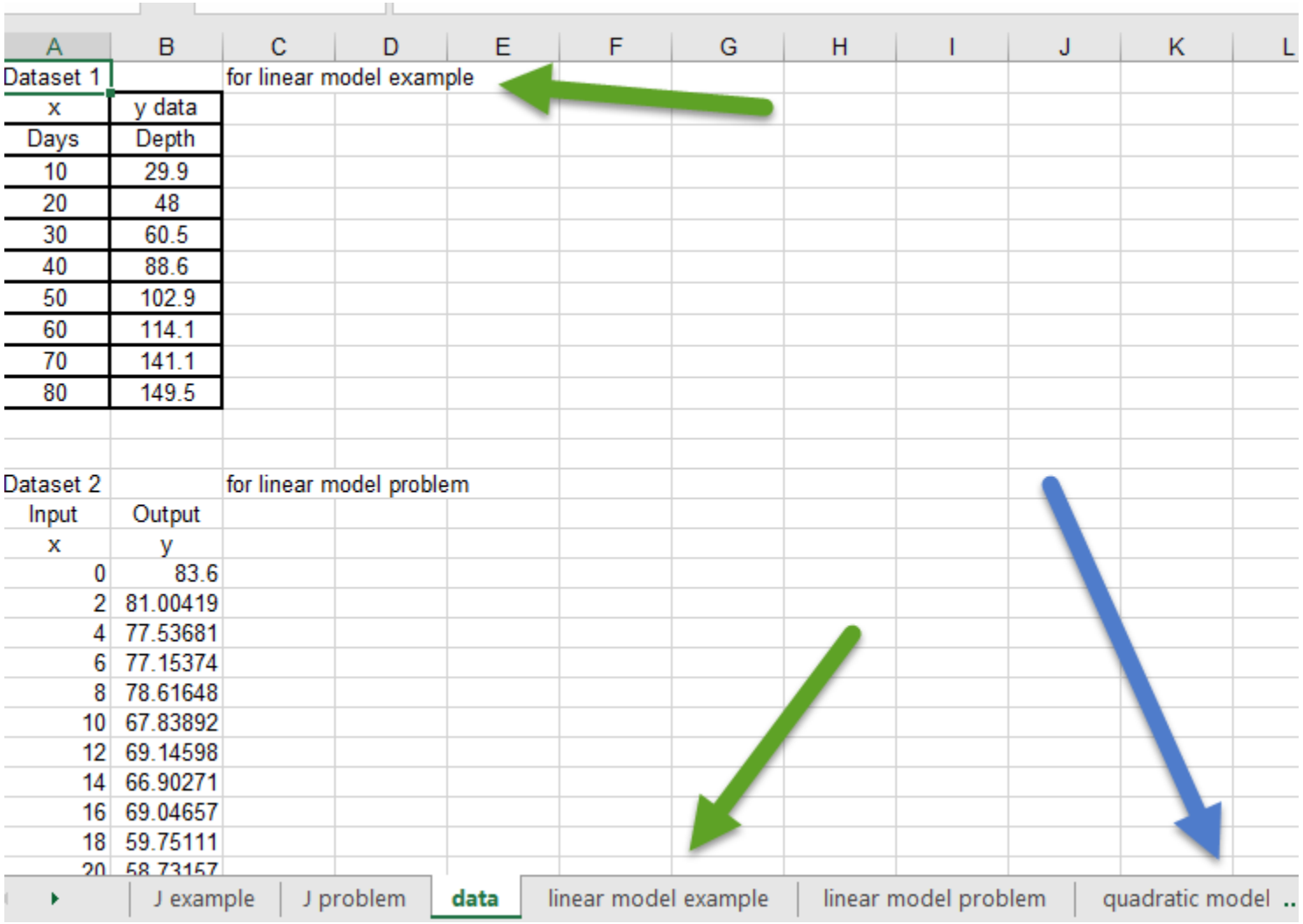


Next step

Give the students a spreadsheet already set up to make it easy for them to start estimating the parameters of the formula from data.

By the end of the course, they are expected to start with a blank spreadsheet, copy the data into it and create all the columns and formulas themselves, as well as the graphs and find good parameter values.

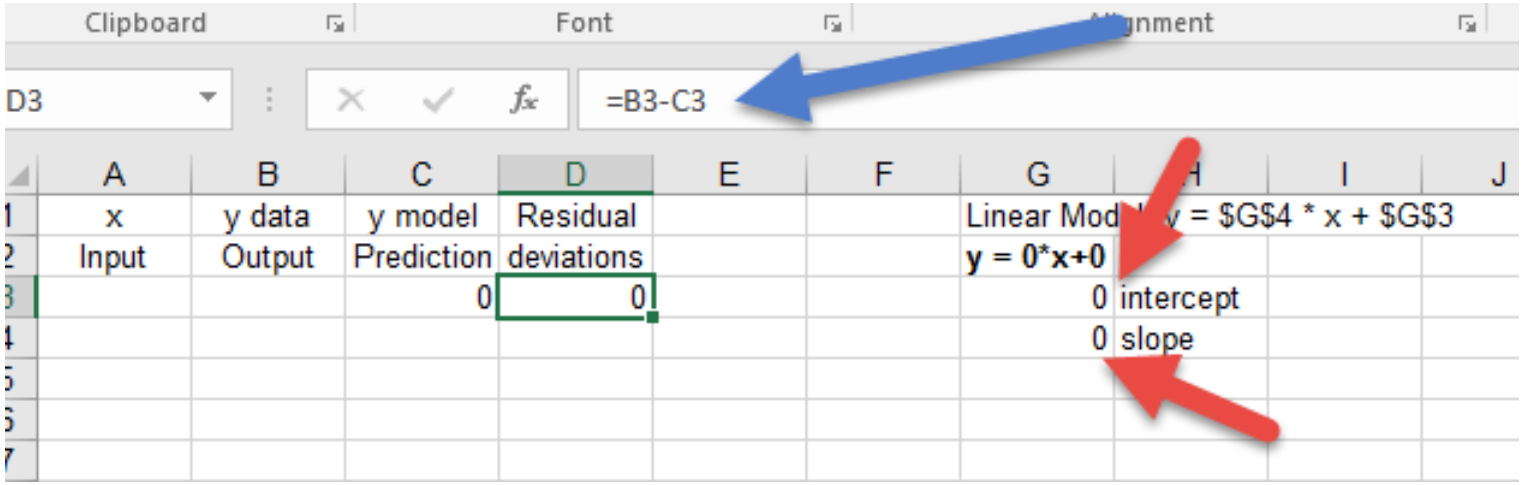
Page 1 of the workbook for the first lesson in fitting data



Dataset 1		for linear model example
x	y data	
Days	Depth	
10	29.9	
20	48	
30	60.5	
40	88.6	
50	102.9	
60	114.1	
70	141.1	
80	149.5	

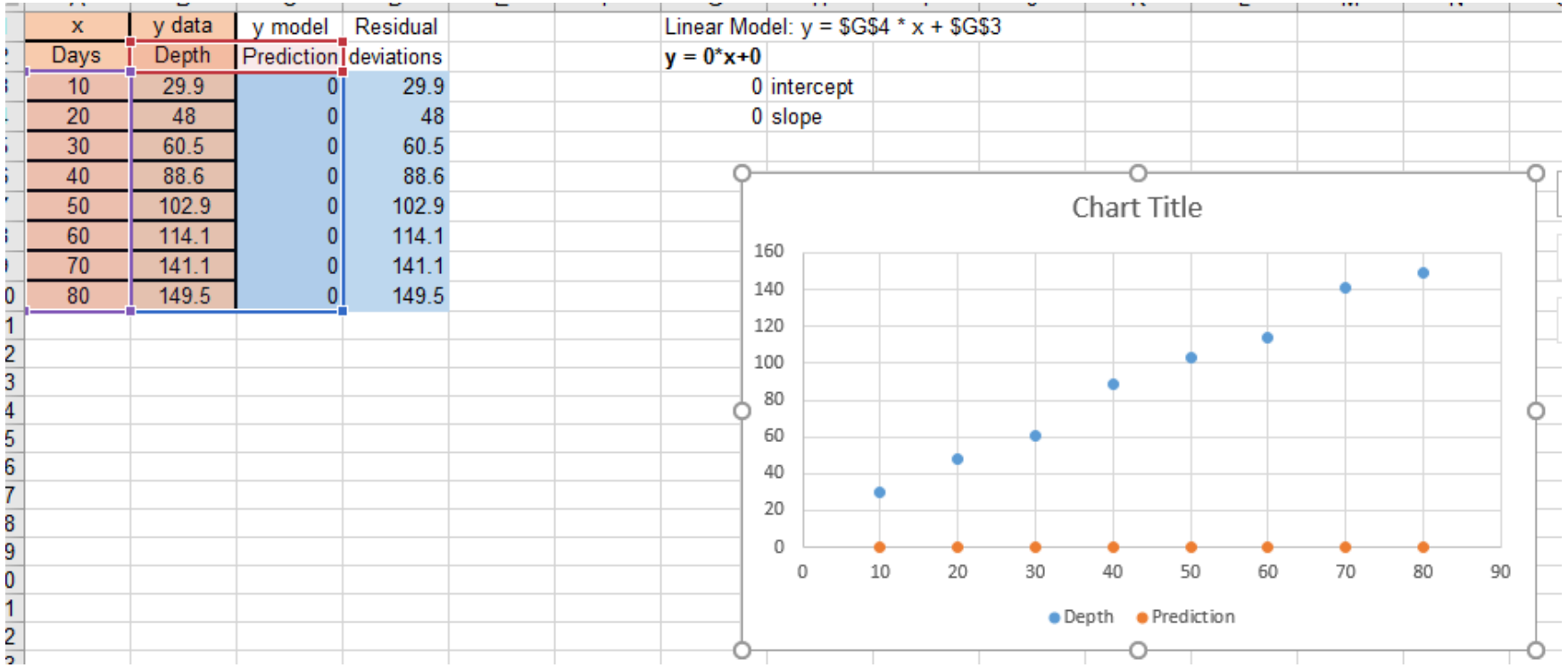
Dataset 2		for linear model problem
Input	Output	
x	y	
0	83.6	
2	81.00419	
4	77.53681	
6	77.15374	
8	78.61648	
10	67.83892	
12	69.14598	
14	66.90271	
16	69.04657	
18	59.75111	
20	58.73157	

Page 3 of the workbook, as provided to students



- The labels of the columns
- The formulas
- The places to enter the parameter values (Red arrows here)
- Labels in cells G1 and G2 that are more elaborate than we will expect students to do, but useful in the beginning.

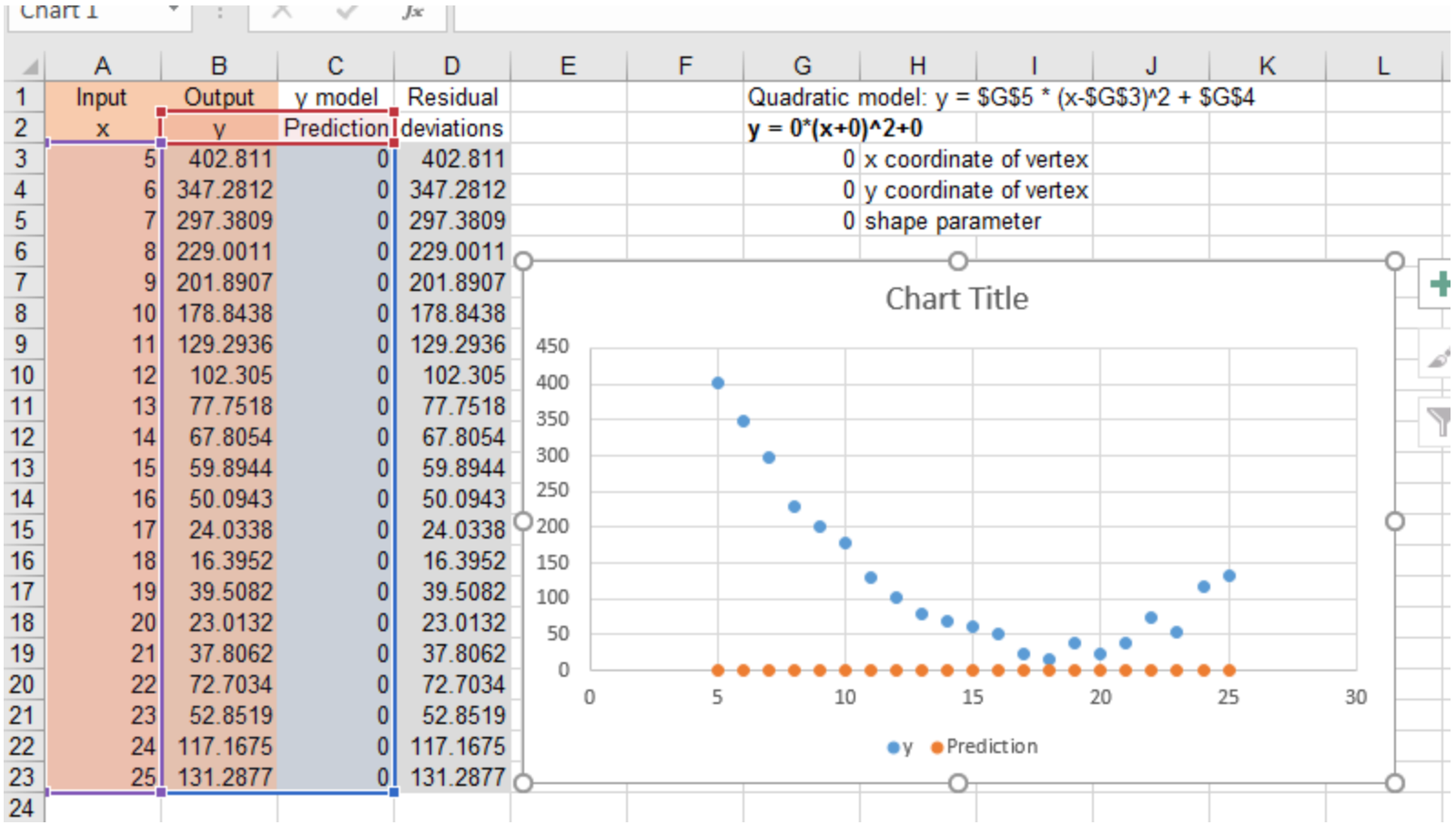
Page 2 of workbook, as students started



x	y data	y model	Residual
Days	Depth	Prediction	deviations
10	29.9	0	29.9
20	48	0	48
30	60.5	0	60.5
40	88.6	0	88.6
50	102.9	0	102.9
60	114.1	0	114.1
70	141.1	0	141.1
80	149.5	0	149.5

- They copied data to the sheet
- They “pulled down” the formulas for the prediction and the deviations
- They made a graph
- Then they estimated the slope and intercept

Page 5 of workbook. Note the parameterization of the quadratic model.



	A	B	C	D
1	Input	Output	y model	Residual
2	x	y	Prediction	deviations
3	5	402.811	0	402.811
4	6	347.2812	0	347.2812
5	7	297.3809	0	297.3809
6	8	229.0011	0	229.0011
7	9	201.8907	0	201.8907
8	10	178.8438	0	178.8438
9	11	129.2936	0	129.2936
10	12	102.305	0	102.305
11	13	77.7518	0	77.7518
12	14	67.8054	0	67.8054
13	15	59.8944	0	59.8944
14	16	50.0943	0	50.0943
15	17	24.0338	0	24.0338
16	18	16.3952	0	16.3952
17	19	39.5082	0	39.5082
18	20	23.0132	0	23.0132
19	21	37.8062	0	37.8062
20	22	72.7034	0	72.7034
21	23	52.8519	0	52.8519
22	24	117.1675	0	117.1675
23	25	131.2877	0	131.2877
24				

Quadratic model: y = G5 * (x-G3)^2 + G4

y = 0*(x+0)^2+0

0 x coordinate of vertex

0 y coordinate of vertex

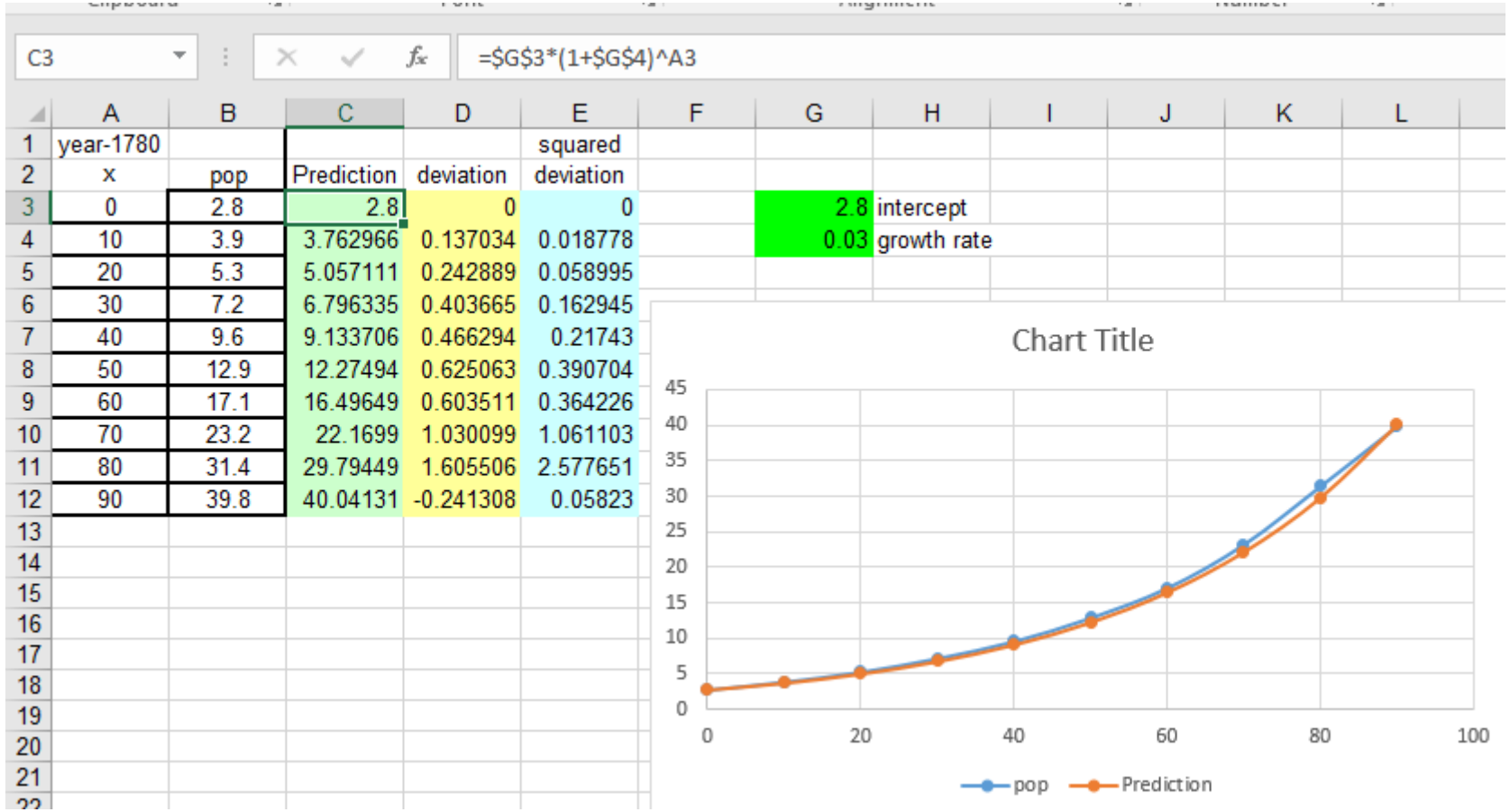
0 shape parameter

Later in the course.

For these data, chose whether to fit a linear, quadratic, or exponential model. Start from a blank spreadsheet.

Year	Population (millions)
1780	2.8
1790	3.9
1800	5.3
1810	7.2
1820	9.6
1830	12.9
1840	17.1
1850	23.2
1860	31.4
1870	39.8

Note the adjusted x-values and the growth-rate parameterization.



	A	B	C	D	E	F	G	H	I	J	K	L
1	year-1780				squared							
2	x	pop	Prediction	deviation	deviation							
3	0	2.8	2.8	0	0		2.8	intercept				
4	10	3.9	3.762966	0.137034	0.018778		0.03	growth rate				
5	20	5.3	5.057111	0.242889	0.058995							
6	30	7.2	6.796335	0.403665	0.162945							
7	40	9.6	9.133706	0.466294	0.21743							
8	50	12.9	12.27494	0.625063	0.390704							
9	60	17.1	16.49649	0.603511	0.364226							
10	70	23.2	22.1699	1.030099	1.061103							
11	80	31.4	29.79449	1.605506	2.577651							
12	90	39.8	40.04131	-0.241308	0.05823							
13												
14												
15												
16												
17												
18												
19												
20												
21												

Why spreadsheets?

- Requires active participation.
- Different type of engagement in this from working with algebra (symbolic manipulation.) Usually students have a better attitude about trying to make it make sense.
- Generally speaking, everyone's product looks good – considerably more so than when doing work by hand.

Why spreadsheets?

- Making successive adjustments to improve their estimates and their solutions shows students a different (and useful) aspect of quantitative thinking than in previous math courses.

- Students successfully do considerably more complex problems than they expect to be able to do.

See
http://www.austincc.edu/mparker/talks/
or mparker@austincc.edu

Includes these slides, a longer set of slides with more details, and links to some materials.