Tutoring Non-Standard Math Problems

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May 9, 2005

The more we try to make math accessible, interesting, and useful to students in various fields, the more challenging it is to teach and to tutor students in these courses. In many ways, tutoring them is the harder of the two tasks. At least the teacher gets to plan the curriculum so they know what the students have already learned and what they are emphasizing in each lesson. The tutors often have to figure out these things “on the fly” from questions that the students ask.

I hope that, by the end of the hour today, we will all have learned about some different perspectives from each other.

Challenges

• Presenting topics from four perspectives: verbal, analytical, graphical, numerical. When a teacher is doing this, it may be challenging for a tutor to know how the student is expected to solve a particular problem.

• Using technology tools. How can a tutor keep up with how to do all these problems on several kinds of graphing calculators and different software, such as spreadsheets or statistical software?

• Elective math courses. Tutors mostly took the standard math sequence in school, so usually they didn’t take the various elective math courses that were available when they were in college. Moreover, there are more of these courses available now than even ten years ago, and a wider variety of topics in them. How should tutors prepare to tutor these?

• So many students need tutoring that you can’t take much time with any one student.

Recommendations:

• When a student asks you to work a problem, if possible, work through an example from the textbook with her first. That promotes the student’s skills in independent learning and helps you determine the emphasis of that portion of the course.

• Preferably, don’t show students shortcuts.

• Many times the teacher is interested in having the students write explanations, sentences, definitions of variables, etc. Support that.
• Especially in an elective math course, remember that the student may not have had the same background that you had before they started this topic.

• Encourage the students in the same class to work together and explain material to each other. Encourage the lab managers/administration to set up situations where tutors can work with multiple students in one class at the same time.

• Many of our courses have “thought questions” that address some of the higher-order analytical skills. Be positive with the students about the importance of these higher-order analytical skills and work out some system which enables you to continue learning about these in the subjects you tutor.

What are some of these higher-order skills addressed in our various courses?

MATH 1342  Elementary Statistics:

It is not uncommon for students to say, “This is elementary statistics. You can’t possibly mean I have to do more than do the computations.”

Yes, we do mean that students have to learn more than computations! On any given problem, however, the student will probably be more focused on getting the computation right than on interpreting it correctly. They need quite a bit of encouragement to believe they can do more.

Following are some of the typical extensions of the usual computations that students find difficult. (This is not meant to be a comprehensive list.)

<table>
<thead>
<tr>
<th>Topic</th>
<th>Computational skill.</th>
<th>Related analytical skills. Definitely required in MATH 1342.</th>
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</table>
| Graphical representation of data | Make a histogram.    | 1. Make a stemplot, including splitting stems.  
2. From a stemplot or histogram, list the values or find the median  
3. From the stemplot or histogram, estimate the center and variability. |
| Numerical summaries of data | Find the mean, median, standard deviation, quartiles, IQR, etc. | 1. Determine which type of summary is most appropriate for a dataset by thinking about whether the shape is fairly symmetric or skewed.  
2. Answer questions such as: “If we have the mean and median of the price of homes in a city are $89,000 and $134,500, which number is which average and why?” |
| **Regression descriptive** | **Find the regression line.** | 1. Interpret the slope.  
2. Answer questions which require interpreting the slope.  
3. Answer questions that require using r-squared. (In simple regression, r-squared is the percent variation in y that is explained by the variation in x.)  
4. Make and interpret residual plots. (To support a linear relationship, the residuals must not show a pattern, but be fairly scattered around the zero line and show about the same variability across the entire range of x-values.) |
|----------------------------|-------------------------------|--------------------------------------------------|
| **Confidence intervals**   | **Find a 99% confidence interval from the given data.** | 1. Write an interpretation of the confidence interval. (Mention that it is for the population parameter, and that it is “confidence” not “probability.”)  
2. Answer a question about the relative sizes of the margin of error for 90%, 95%, and 99% confidence.  
3. If we want a confidence interval for a subgroup of the population, like only the women, but from the same sample, determine whether the margin of error will be larger or smaller. |
| **Hypothesis testing**     | **Is the result significant? Yes or no?** | 1. Draw a picture of the sampling dist’n of the test statistic, shade in the area that represents the p-value, and describe what it means.  
2. Find the p-value and explain what it means. |
| **Hypothesis testing**     | **Choosing the significance level arbitrarily.** | 1. The significance level should be chosen before looking at the data.  
2. In reality, people should choose the significance level by analyzing the relative importance of Type I and Type II errors. |
| **Defining parameters**    | **Say something sensible in conclusion about the question.** | 1. Notice that both hypotheses have the SAME parameters and that they are UNKNOWN values. Be able to state clearly what population each of those unknown parameters comes from, as well as identifying the parameter by name (mean or proportion.)  
2. Then state the conclusion so that it clearly relates to the unknown parameters. |
| Choose which statistical technique to use for a given problem. | Use whichever method is covered in this section. Or Look at “key words” in the problem. | 1. Use the statement of the problem to decide.  
2. Is there one unknown parameter or two?  
3. Think about what the original data looked like. (categorical or quantitative leads to techniques for proportions or means, in 1342.)  
4. Was the design for data collection matched pairs?  

| Understand the assumptions needed for a statistical technique to be used. | Use whichever method is covered in this section without thinking of the assumptions. | State the assumptions and think about whether you can use the data to assess whether they are met. If not, what questions would you have to ask the data collector to determine whether they were met?  

| Regression inference. Intervals for a prediction | Understand why the prediction interval is longer than the confidence interval for the population mean of the y’s when x = the given value. |  

| Regression inference. Residual plots | Use the software to produce all standard residual plots and show them. | Think about how we need to use the residuals to check  
1. Whether the scatter around the line is random or has a pattern. (residuals vs. explanatory variable.)  
2. Whether the scatter around the line has the same variability for all the x-values. (residuals vs. explanatory variable.)  
3. Whether the residuals appear to be reasonably normally distributed. (histogram or stemplot of residuals)  
(Optional: Understand the meaning of the other standard residual plots produced automatically by MINITAB, even when not covered in our 1342 course.) |

**MATH 1350/1351**

If you will tutor students in this course, it is particularly important to read the Preface to the text – at least the first few pages – in order to understand how to best support the students.

This course is for prospective teachers of elementary and middle school and the point is to teach students multiple ways of understanding the concepts taught at those levels. It is particularly important for the students to learn to use manipulatives to illustrate the concepts of arithmetic and prealgebra and to use drawing/construction to illuminate the concepts of geometry.
Many of these students are not mathematically adept, so they need continual encouragement that it is important to learn about these multiple ways of understanding. Encourage them to think of the way that is most comfortable for them as a base from which they can build additional understanding.

This course is taught from a constructionist perspective, where the students explore the ideas on their own and only at the end, are presented with a summary of the important mathematical ideas covered.

MATH 1333 Mathematics for Measurement

I think of this course as “math for practical arts” as distinguished from “math for liberal arts.” We take the same (large) set of possible topics as for 1332 and choose a somewhat different subset of topics to include, with significant difference in the emphasis. Topics include modeling, applied trigonometry, and some statistics.

General emphases:

1. Students learn to check their work in ways other than “look in the back of the book.” (In the real world, there isn’t a “back of the book.”)

2. Students learn to graph functions (which we call formulas in this course) using a spreadsheet. These include a variety of formulas, including many non-linear formulas. They use a calculator and spreadsheet both to find values. That provides a method of checking their work. From these graphs they learn to find approximate solutions for “solve for x, given y” as well as “solve for y, given x.”

3. Students increase their skills in reading problems and writing answers that include sentences of explanation as well as numbers.

4. Students learn “technical drawing” and can solve applied geometry / trig problems using a careful diagram.

Trigonometry and measurement topics:

1. Students learn right-triangle applied trigonometry and general-triangle applied trigonometry.

2. Students learn the graphs of the sine and cosine formulas with degrees as input (no radians) and use these graphs and their calculator to solve equations such as “Solve $\sin A = -0.3678$ on $-90^0 \leq A \leq 380^0$.” This is mainly used to support the solution of general triangles using trigonometry.

3. Some students algebra skills are barely adequate for the most advanced trig we do (Law of Cosines.) Students will ALWAYS use a good drawing to check their work and can avoid using the Law of Cosines completely with some, but not much, grade penalty since they can simply solve the problems using a careful diagram.
4. Students learn to think of approximate numbers (and measured numbers) as intervals rather than as points on a number line. For most of the course, these are investigated using rounded numbers, so that the end points of the interval are precise numbers. But the end of the course, they learn about summarizing approximate numbers using standard deviation, and multiples of the standard deviation. They use Excel to compute the standard deviation.

5. Students learn to use significant digits in calculations and also learn to use the “error propagation” method, where we find the largest and smallest possible computed values based on rounded input values.

6. Students learn about the difficulties of communicating accuracy via significant digits.

7. Students learn about the “measurement sensitivity” of a formula at a given input value. (This is the slope of the graph of the formula at that point – which we do by thinking of the secant line between two points rather than using the limiting process to get the tangent line.) As a result, they can look at the graph of a formula and say where the measurement sensitivity will be the greatest and least.

8. Students learn to compute and interpret the measurement sensitivity in words and use it to determine how accurately the input value must be to achieve a desired accuracy of the output value.

9. Students learn that we can reduce variability in measurements by taking repeated independent measurements and averaging.

Modeling topics:

By the end of the course, students are able to take data that does not exactly fit a straight line or other exact model and to use a spreadsheet to find a good model for the data. They can use residual plots to assess the fit of the model and use the model to make “forward” and “backward” predictions. Because these students do not have strong algebra skills (and aren’t expected to develop them in this course) this requires careful review of their previous algebra skills with straight lines.

Students learn to work applied linear modeling problems (exact lines) but are not necessarily be adept at all the algebra of the various forms of the equation of a line. They

1. Choose the appropriate “output variable” and use it in the position of the y-variable.

2. Modify the x-variable if needed, to something like \( t = \) years since 1990. They will also understand that the problem can be worked without doing that, but why it is useful to do it. (To make the interpretations of the intercept useful and to simplify computations.) They will also understand that it isn’t critical exactly what the beginning x-value is.
3. Use the slope formula \( m = \frac{y_2 - y_1}{x_2 - x_1} \) to find the slope given two points.

4. Use the point-slope formula \( y - y_0 = m(x - x_0) \) to find the equation of a line (called the formula of the line.)

5. Interpret the slope and intercept, using the appropriate names for the variables and the appropriate units for the numbers. (feet, miles per hour, etc.)

6. Plot the line using a spreadsheet.

7. Use the spreadsheet to find a \( y \), given the \( x \), and also to find an \( x \), given a \( y \).

8. Use the formula for the line to find a \( y \), given the \( x \).

9. (Optional: use the formula for the line to find an \( x \), given a \( y \).)

In extension of the linear modeling problems, they learn to use the spreadsheet to graph a formula that is not a line.

1. Use the spreadsheet to find a \( y \), given the \( x \), and also to find an \( x \), given a \( y \).

2. Use the formula for the line to find a \( y \), given the \( x \).

3. Use the spreadsheet to plug in additional values in order to give a more precise answer to the question of “find an \( x \), given a \( y \).”

In another extension, students use “trendline” on a spreadsheet to find the linear regression line that best fits a given dataset. They then do the usual predictions. (Predict \( y \), given \( x \). Find the \( x \)-value, given a \( y \).)

Students use residual plots (residuals versus the input variable) to determine whether the linear pattern fits well or whether there is another pattern in the data. If there is another pattern, they use other trendlines available in Excel (and residual plots) to find a better-fitting trendline.

**MATH 1332 College Mathematics**

As you know, we have two different textbooks with different emphases for this course. I haven’t taught out of these particular textbooks so I don’t know details chapter by chapter. I do know, however, from my previous experiences with the course, that the same overall ideas discussed for the other courses at this level apply to this course as well.