Elementary Algebra
Review for Exam 3

Solve the problem.

1) After receiving a discount of 15% on its bulk order of typewriter ribbons, John's Office Supply pays $5882. What was the price of the order before the discount? Round to the nearest dollar if necessary."

Multiply.

2) $(-8x^4)(2x^5)(-5x^3)$

3) $-5x^6(-8x^5 + 11x^4)$

4) $(3x + 9)(x + 6)$

5) $(3x - 4)(x - 8)$

6) $(3x^2 + 3)(4x - 6)$

7) $(6p + 7)(6p - 7)$

8) $(w - 7)^2$

9) $(x^2 - 5)^2$

10) $(xy + 7)(8xy - 4)$

11) $(2x + 7y)^2$

12) $(x - 3)(7x^2 + x + 8)$

Evaluate as requested.

13) Evaluate the polynomial $-2x^2 - y^2 + xy$ for $x = -2$ and $y = 5$.

Solve the problem.

14) The polynomial $0.041h - 0.018A - 2.69$ can be used to estimate the lung capacity, in liters, of a female with height h cm and age A years. Find the lung capacity of a 20-year-old woman who is 179 cm tall. Round to the nearest liter.

Determine the coefficient and the degree of each term in the polynomial. Then state the degree of the polynomial.

15) a) $8w^5 + 5w^3 - w$

  b) $-4xyz^5 + 15x^5y^4 + xz^4$

Find the total area of all shaded rectangles.

16)
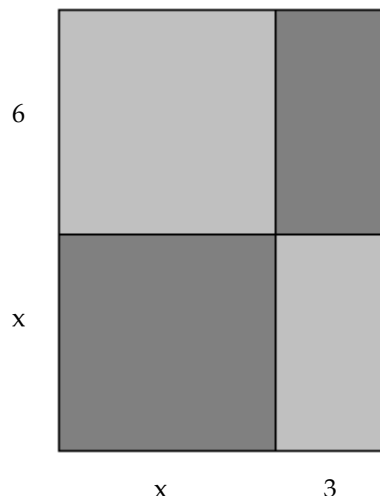
Add or subtract, as indicated.

17) a) $(x^2 - 8xy + 7y^2) + (7x^2 - 9xy - y^2) + (-3x^2 + xy - y^2)$

  b) $(3w^4 - 3wz + 9wz^2) - (6w^4 + 2wz - 5wz^2)$

Divide.

18) $(35x^7 - 49x^4 + 14x^2) \div (7x^2)$

19) $(x^2 + 13x + 34) \div (x + 6)$

20) $(15x^2 + 7x - 2) \div (5x - 1)$

21) $\dfrac{x^3 + 125}{x + 5}$

Express using positive exponents. Then, if possible, simplify.

22) a) 4^{-2} b) $\left(\dfrac{2}{3}\right)^{-3}$

  c) $\dfrac{1}{x^{-4}}$ d) $\left(\dfrac{4}{u}\right)^{-3}$

Simplify. Do not use negative exponents in your answer.

23) a) $y^{-8} \cdot y^{-4}$ b) $(x^{-5}y^{-3})(x^9y^{-7})$

  c) $3^{-3} \cdot 3^4$ d) $(x^9y^{-8}z^{-5})(x^{-2}y^{-5}z^{10})$

24) a) $\dfrac{p^4}{p^{-5}}$ b) $\dfrac{z^{-8}}{z^{-4}}$ c) $\dfrac{7x^{-6}}{y^{-2}z^3}$

25) a) $(x^{-7})^3$ b) $(2xy)^{-4}$

c) $(x^{-2}y^{-3})^{-6}$ d) $(2m^4n^{-5})^3$

26) a) $\left(\dfrac{x^3}{3}\right)^{-2}$ b) $\left(\dfrac{x^5y^5}{wz^6}\right)^{-3}$

c) $\left(\dfrac{s^{-4}}{t^{-3}}\right)^6$ d) $\left(\dfrac{8x^{-3}}{3y^{-5}z}\right)^0$

Convert to decimal notation.

27) a) 2.84×10^7 b) 8.441×10^{-5}

Convert to scientific notation.

28) a) 33,000,000 b) 0.000080517

Perform the indicated operation. Write the answer in scientific notation.

29) a) $(2.6 \times 10^{-5})(1.1 \times 10^{-9})$

b) $(4.7 \times 10^{-3})(7.5 \times 10^7)$

c) $\dfrac{9.12 \times 10^2}{2.4 \times 10^6}$

d) $\dfrac{8.04 \times 10^{-8}}{2 \times 10^{-4}}$

Solve the problem. Express the answer in scientific notation to two decimals unless requested otherwise.

30) The earth is approximately 92,900,000 miles from the sun. If 1 mile $= 1.61 \times 10^3$ m, what is the distance to the sun in meters?

Provide an appropriate response.

31) For what values, if any, of $x \left(x \neq 0\right)$ will x^{-3} be a negative number?

32) Explain the different meanings of the "–" symbol for $5 - 2$, -3, and 3^{-2}.

Factor out the greatest common factor.

33) $4x^{12}y^8 - 8x^9y^7 + 12x^6y^5 - 16x^3y^5$

Determine whether the following is a perfect–square trinomial.

34) a) $x^2 + 18x + 81$

b) $x^2 - 20x + 400$

c) $x^2 - 14xy + 49y^2$

Factor completely. If the polynomial is prime, state this.

35) $x^2 + 3x - 18$

36) $x^2 - 8x - 33$

37) $x^2 - 24 + 2x$

38) $3x^4 - 21x^3 + 30x^2$

39) $24x^3 - 62x^2 + 40x$

40) $y^2 - \dfrac{2}{3}y + \dfrac{1}{9}$

41) $15y^2 + 16y + 4$

42) $9z^2 + 6z - 8$

43) $9x^2 - 39xy - 30y^2$

44) $x^2 - 6x + 36$

45) $9 - 30x + 25x^2$

46) $25x^2 - 60x + 36$

47) $128x^2 - 192x + 72$

48) $2x^3 - 4x^2 + 2x$

49) $12x^2 + 17x + 6$

50) $y^2 - 36$

51) $81p^2 - 4q^2$

52) $16x^2 + 25$

53) $4x^2 - 36$

54) $49y^4 - 64$

55) $x^4 - 81$

56) $2x^3 - 12x^2 + 8x - 48$

57) $6x^2 - 15x - 8xy + 20y$

Solve the equation.

58) $(x - 7)(x + 6) = 0$

59) $(11 - 0.8x)(5 - 0.2x) = 0$
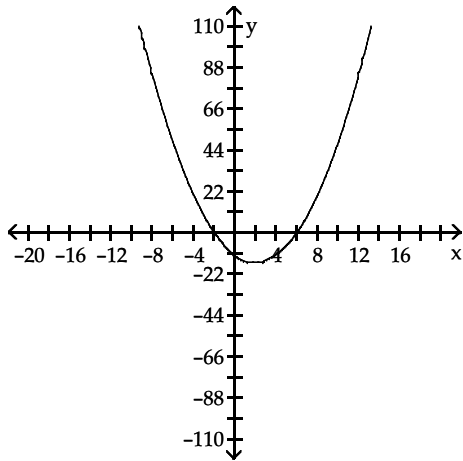
60) $x^2 - x = 72$

61) $16k^2 - 25 = 0$

62) $4x^2 = 5x$

63) $x^2 + 3x - 88 = 0$

64) $(x + 4)(x - 7) = -10$

Find the x–intercepts for the graph of the equation.

65) $y = x^2 - 4x - 12$

Solve the problem.

66) The length of a rectangular picture frame is 9 cm more than the width. The area inside the frame is 90 square cm. Find the dimensions of the frame.

67) One leg of a right triangle is 6 cm shorter than the other leg. The length of the hypotenuse is 30 cm. Find the length of the shorter leg.

68) A 10–ft ladder is leaning against a building. If the bottom of the ladder is 6 ft from the base of the building, how high does the ladder reach?

69) In a sports league of n teams in which each team plays every other team twice, the total number N of games to be played is given by N $= n^2 - n$. How many teams are in a softball league if the total number of games played is 56?

70) If an object is propelled upward from a height of 96 feet at an initial velocity of 80 feet per second, then its height after t seconds is given by the equation h $= -16t^2 + 80t + 96$, where h is in feet. After how many seconds will the object reach a height of 196 feet?

Provide an appropriate response.

71) Explain the difference between a factor and a multiple of a number.

72) Explain the error in the following:
$x^2 + 2x - 15 = (x - 5)(x + 3)$

73) What is the difference between a trinomial and a quadratic equation?

74) Will there always be two different solutions when the principle of zero products is used to solve a quadratic equation? Why or why not?

75) What is wrong with solving $x^2 = 6x$ by dividing both sides of the equation by x?

76) Explain the error in the following:
$x^2 - y^2 = (x - y)^2$

77) Explain the error in the following:
$25x^2 - 100 = (5x + 10)(5x - 10)$

78) Explain the error in the following:
$x^2 + 16 = (x + 4)(x - 4)$

79) How could you use factoring to convince someone that $x^2 - y^2 \neq (x - y)^2$?

80) Explain the error in the following, assuming you were to factor completely:
$81x^4 - 1 = (9x^2 + 1)(9x^2 - 1)$

1) $6940

2) $80x^{12}$

3) $40x^{11} - 55x^{10}$

4) $3x^2 + 27x + 54$

5) $3x^2 - 28x + 32$

6) $12x^3 - 18x^2 + 12x - 18$

7) $36p^2 - 49$

8) $w^2 - 14w + 49$

9) $x^4 - 10x^2 + 25$

10) $8x^2y^2 + 52xy - 28$

11) $4x^2 + 28xy + 49y^2$

12) $7x^3 - 20x^2 + 5x - 24$

13) -43

14) 4 liters

15) a) Coefficients: 8, 5, –1;
    degrees: 5, 3, 1
    degree of polynomial: 8
   b) Coefficients: –4, 15, 1
    degrees: 7, 9, 5
    degree of polynomial: 9

16) $x^2 + 9x + 18$

17) a) $5x^2 - 16xy + 5y^2$
   b) $-3w^4 - 5wz + 14wz^2$

18) $5x^5 - 7x^2 + 2$

19) $x + 7 - \dfrac{8}{x + 6}$

20) $3x + 2$

21) $x^2 - 5x + 25$

22) a) $\dfrac{1}{16}$ b) $\dfrac{8}{27}$

    c) x^4 d) $\dfrac{u^3}{64}$

23) a) $\dfrac{1}{y^{12}}$ b) $\dfrac{x^4}{y^{10}}$

    c) 3   d) $\dfrac{x^7z^5}{y^{13}}$

24) a) p^9 b) $\dfrac{1}{z^4}$ c) $\dfrac{7y^2}{x^6z^3}$

25) a) $\dfrac{1}{x^{21}}$ b) $\dfrac{1}{16x^4y^4}$

    c) $x^{12}y^{18}$ d) $\dfrac{8m^{12}}{n^{12}}$

26) a) $\dfrac{9}{x^6}$ b) $\dfrac{w^3z^{18}}{x^{15}y^{15}}$

    c) $\dfrac{t^{18}}{s^{24}}$ d) 1

27) a) 28,400,000   b) 0.00008441

28) a) 3.3×10^7 b) 8.0517×10^{-5}

29) a) 2.86×10^{-14} b) 3.525×10^5
    c) 3.8×10^{-4} d) 4.02×10^{-4}

30) 1.50×10^{11} m

31) If x is negative, the answer will be negative.

32) Subtraction; negative, or opposite; reciprocal

33) $4x^3y^5(x^9y^3 - 2x^6y^2 + 3x^3 - 4)$

34) a) Yes   b) No   c) Yes

35) $(x + 6)(x - 3)$

36) $(x + 3)(x - 11)$

37) $(x + 6)(x - 4)$

38) $3x^2(x - 5)(x - 2)$

39) $2x(4x - 5)(3x - 4)$

40) $\left(y - \dfrac{1}{3}\right)^2$

41) $(3y + 2)(5y + 2)$

42) $(3z - 2)(3z + 4)$

43) $3(3x + 2y)(x - 5y)$

44) Prime

45) $(5x - 3)^2$

46) $(5x - 6)^2$

47) $8(4x - 3)^2$

48) $2x(x - 1)^2$

49) $(4x + 3)(3x + 2)$

50) $(y + 6)(y - 6)$

51) $(9p + 2q)(9p - 2q)$

52) Prime

53) $4(x+3)(x-3)$

54) $(7y^2 + 8)(7y^2 - 8)$

55) $(x^2 + 9)(x - 3)(x + 3)$

56) $2(x - 6)(x^2 + 4)$

57) $(2x - 5)(3x - 4y)$

58) 7, –6

59) 13.75, 25

60) –8, 9

61) $\dfrac{5}{4}, -\dfrac{5}{4}$

62) $\dfrac{5}{4}, 0$

63) –11, 8

64) –3, 6

65) (–2, 0), (6, 0)
66) width = 6 cm, length = 15cm
67) 18 cm
68) 8 ft
69) 8
70) 2.5 sec
71) A factor is a number in a product while a multiple of a number is a product of it and some natural number. For example, 6 = 2 • 3, so 2 and 3 are FACTORS of 6; 18 = 3 x 6, so 18 is a MULTIPLE of 6.
72) The factorization yields a middle term of –2x rather than 2x:
$x^2 + 2x - 15 = (x + 5)(x - 3)$
73) A trinomial is the sum of three monomials. It is not an equation. A quadratic equation is an equation of the form $ax^2 + bx + c = 0$, a ≠ 0.
74) No; whenever $ax^2 + bx + c$ is a trinomial square, then $x^2 + bx + c$ has only one solution.
75) One solution of the equation is 0. Dividing both sides of the equation by x, leaving the solution x = 6, is equivalent to dividing by 0.
76) $x^2 - y^2$ is a difference of squares, not a binomial square.
$x^2 - y^2 = (x + y)(x - y)$
77) The largest factor has not been factored out:
$25x^2 - 100 = 25(x + 2)(x - 2)$
78) The polynomial $x^2 + 16$ is not a difference of squares. Since $x^2 + 16$ is a sum of squares with no common factor, it cannot be factored.
79) $x^2 - y^2 = (x + y)(x - y) ≠ (x - y)^2$
80) The polynomial is not factored completely since $9x^2 - 1$ can be factored further:
$81x^4 - 1 = (9x^2 + 1)(3x + 1)(3x - 1)$