MATH 2413 - Calculus I - Prerequisite Review

Give exact answers unless a problem specifies otherwise.

1. Rationalize the numerator and simplify: $\dfrac{\sqrt{x+5}-\sqrt{x}}{10}$

2. Simplify and give your answer in simplified radical form: $\dfrac{x^{2/3}\cdot x^{-3/5}}{(x^{-5/6})^2}$

3. Simplify: $\dfrac{x^3-8}{x^3-4x} \div \dfrac{3x^2+2x-8}{x^2+4x+4}$

4. Simplify: $\dfrac{2x-3}{x^2-3x-4} - \dfrac{3x-5}{x^2-8x+16}$

5. Give the slope-intercept form of the equation of the line perpendicular to $5x - 3y = 8$ and passing through $(-4,7)$.

6. Give the slope of the line that passes through the points P and Q on the graph of $f(x) = \cos x$ if P has an x-coordinate of $\dfrac{2\pi}{3}$ and Q has an x-coordinate of $\dfrac{5\pi}{4}$.

7. A tailor works 8 hours a day, 6 days a week. He divides his time between making shirts and making coats. It takes him 4 hours to make a shirt and 6 hours to make a coat. Let x be the number of shirts the tailor makes in a week and let y be the number of coats the tailor makes in a week. (a) Find a formula for y as a function of x. Interpret (b) the slope and (c) the y intercept in this context.

8. Evaluate without a calculator: (a) $\sin\left(\frac{7\pi}{3}\right)$ (b) $\tan\left(\frac{7\pi}{4}\right)$ (c) $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ (d) $\cos\left(\sin^{-1}(-1)\right)$

9. Use trig identities to convert the first expression to the second, if possible:

(a) $\dfrac{\sec x}{\cos x}$, $1 + \tan^2 x$

(b) $\sin t(\csc t - \sin t)$, $\cos^2 t$

(c) $\cos\left(x + \dfrac{\pi}{2}\right)$, $-\sin x$

(d) $\sin^2 x \cdot \cos^2 x$, $\dfrac{1}{8}(1 - \cos 4x)$

10. Simplify: $\sin(2 \arctan x)$

11. Suppose you are asked to graph $r = f(s)$. (a) Which variable is the dependent variable? (b) If 4 is a solution to the equation $f(s) = 7$, give the coordinates of a point on the graph of f.

12. For $g(x) = \dfrac{|x+4|}{x+4}$, determine (a) $g(-2)$ and (b) $g(-5)$.

13. For $f(x) = \dfrac{5}{2x-3}$, determine $\dfrac{f(x+h)-f(x)}{h}$ and simplify. (When you are finished, the denominator should not have h as one of its factors.)

14. Let $f(x) = \dfrac{7x}{3x^2+5}$. (a) Is $f(x)$ an even function, an odd function, or neither? (b) Describe the symmetry, if any, of the graph of $f(x)$.

15. For $f(x) = \dfrac{3}{x^2+2}$ and $g(x) = 3x - 5$, determine (a) $f(g(x))$ and (b) $g(f(x))$.

16. For $h(x) = \sqrt[3]{x^2 + 9}$, define two functions $f(x)$ and $g(x)$ such that h is the composite of f and g; in other words, decompose h into f and g so that $h(x) = f(g(x))$.

17. For $f(x) = \dfrac{3x-5}{2x+7}$, determine $f^{-1}(x)$.

18. For $g(x) = \dfrac{1}{\sin x}$, determine $g^{-1}(x)$.

19. Give the domain of each of the following. Use interval notation where appropriate.

(a) $f(x) = 0.3x^3 - x^2 + \frac{2}{3}x - 8$

(b) $f(x) = \left|\dfrac{3x-5}{4}\right|$

(c) $f(x) = \dfrac{x^2 - x - 6}{x^2 + 2x - 8}$

(d) $f(x) = \sqrt{2 - 7x}$

(e) $f(x) = \sqrt[3]{4x - 5}$

(f) $f(x) = \log(2x + 5)$

(g) $f(x) = e^{3x+5}$

(h) $f(x) = \tan x$

(i) $f(x) = \arcsin\left(\dfrac{2 - 9x}{5}\right)$

(j) $f(x) = \tan^{-1}x$

(k) $g(x) = \dfrac{2x - 5}{3x^2 - 4x + 5}$

(l) $h(x) = \dfrac{5}{\sqrt[3]{x}} + (\ln(x+3))^2$

(m) $f(x) = \ln(x^2)$

20. (a) Graph $y = \begin{cases} 4 - x & \text{if } x \le 0 \\ 4 - x^2 & \text{if } 0 < x \le 3 \\ -\frac{1}{3}x - 5 & \text{if } x > 3 \end{cases}$

(b) Is this relation a function? Explain.
(c) Is this relation a one-to-one function? Explain.
If we call this relation $f(x)$, determine
(d) $f(3)$ and (e) $f^{-1}(-4)$.

21. Give (a) the x-intercepts and (b) the y-intercept of $f(x) = -9x^2 - 6x + 2$.

22. Suppose $g(x)$ is a 5th degree polynomial with a negative leading coefficient and a negative constant term. True or false:
(a) As $x \to \infty, g(x) \to \infty$.
(b) g has, at most, four x-intercepts.
(c) g has, at most, four relative extrema (peaks and valleys).
(d) The graph of g crosses the y-axis below the x-axis.

23. Describe the graph of $2x^2 + 2y^2 - 6x + 8y + 5 = 0$. (Give sufficient detail so that a classmate could draw the figure from your description.)

24. Graph $f(x) = -\sqrt{25 - x^2}$

25. Give the range of $g(x) = 4 - 3(2^{-x})$

26. Describe the behavior of each of the following:
(a) 5^{-x} as $x \to \infty$
(b) $\log x$ as $x \to 0^+$
(c) $\ln x$ as $x \to \infty$
(d) 4^x as $x \to -\infty$

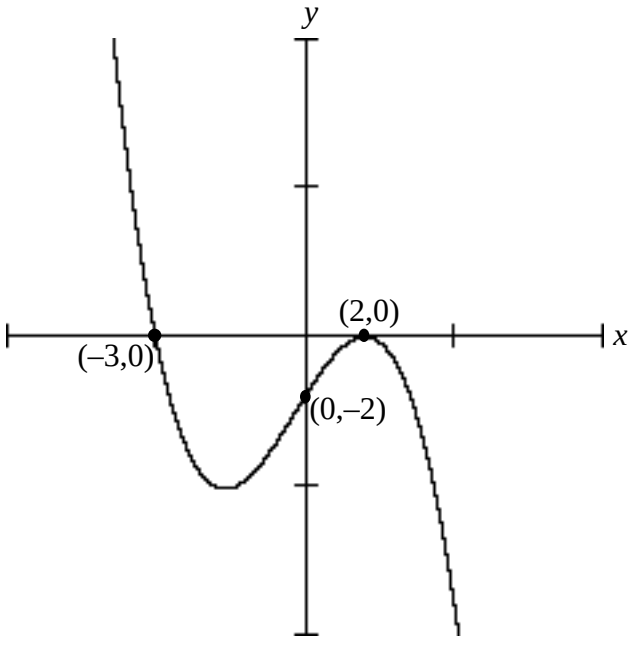
27. (a) For values of x in the interval (0,1), which is larger? x^3 or x^4?

(b) For values of x in the interval (1,2), which is larger? x^3 or x^4?

(c) Describe the behavior of $y = x^{-20}$ as $x \to \infty$

(d) Describe the behavior of $y = x^{20}$ as $x \to -\infty$.

28. Give the formula, in factored form, of the polynomial $p(x)$ whose graph is shown below.



29. Let $f(x) = \dfrac{(2x-3)(x+3)^2}{(x-3)^2(x+1)}$. True or false:

(a) As $x \to \infty, f(x) \to 1$. (b) As $x \to -\infty, f(x) \to 2$. (c) As $x \to 3^+, f(x) \to \infty$.

(d) As $x \to 3^-, f(x) \to \infty$. (e) As $x \to 1^+, f(x) \to \infty$. (f) As $x \to 1^-, f(x) \to -\infty$.

(g) The graph of f goes across the x-axis at $x = \dfrac{3}{2}$.

(h) The graph of f "bounces" against the x-axis at $x = 3$.

30. For $h(x) = -3\sin\left(2x + \dfrac{\pi}{2}\right) + 5$, give (a) the period, (b) the amplitude, (c) the equation of the midline, (d) the amount and direction of the vertical translation, (e) the amount and direction of the horizontal translation, and (f) the range.

31. If $f(x) = \dfrac{3x-7}{2x+5} - \dfrac{3x+4}{2x-3}$ and $g(x) = \dfrac{5}{4x^2+4x-15}$, determine the values of x for which $f(x) = g(x)$.

32. Solve: $\sqrt{x+8} - \sqrt{2x+9} = 1$

33. Solve: $x^{-1/3} = 3x^{-4/3} + 10x^{-7/3}$

34. Solve: $2\left(5^{2x-3}\right) - 4 = 7$ Give both the exact solution and the solution rounded to the nearest thousandth.

35. The populations (in thousands) of two different cities are given by $P_1 = 45(1.038)^t$ and $P_2 = 48(1.025)^t$, where t is the number of years since 2001. In how many years will the populations of the two cities be equal? Round your answer to the nearest hundredth of a year.

36. Determine the x-intercept(s) of $f(x) = \log_2(3x-4) + 1$.

37. Solve: $\log(2x+3) - \log(3x-4) = 2$

38. Solve: $\log_2(x-2) + \log_2 x = 3$

39. Solve for x in $[0,2\pi)$: $2\sin^2 x - \cos x - 1 = 0$

40. Solve for t, rounded to the nearest thousandth, in $[0,2\pi]$: $3\sin^2 t = \cos t$

41. Solve for α, where $0 \le \alpha \le 2\pi$: $\sec^2\alpha - 2\tan\alpha = 4$
Give exact solutions when possible; otherwise round to the nearest thousandth.

42. For $f(x) = \cos x + \sin x$, determine ***all*** of the values of x for which $f(x) = 0$.

In Problems 43-51, solve and give the solutions in interval notation.

43. $2 - 3(4x-5) > 10 - 7x$

44. $|3x-5| + 2 > 9$

45. $2x \le x^2 - 15$

46. $2x^5 - 5x^4 - 14x^3 + 35x^2 > 0$

47. $\dfrac{x-3}{x+5} > \dfrac{x+2}{x-4}$

48. $2^{-x} > 8$

49. $7^x - 1 \le 3$

50. $\log(2x-5) \le 3$

51. $\ln(-3x+5) \le 0$

52. If w is inversely proportional to the square of v and $w = 36$ when $v = 5$, write the formula for w as a function of v.

53. The distance between Austin and Georgetown is 30 miles. Ramon bikes part of the way at an average speed of 20 mi/hr. Then he jogs the rest of the way at an average speed of 6 mi/hr. Let x

be the distance that Ramon rides his bike and write the formula for the function that give the total time for Ramon's trip.

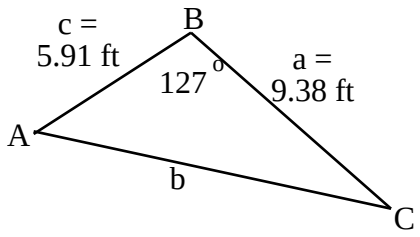
54. An 8 foot long ladder is leaned against the side of a two-story house. Let x be the distance between the bottom of the house and the bottom of the ladder. Write the formula for the function $f(x)$ that gives the height (distance above the ground) at which the top of the ladder touches the houses.

55. A rectangular sheet of cardboard is 8 inches by 10 inches. Squares are cut out of each corner of the cardboard so that the cardboard can be folded to form an open box. Let x be the length of a side of the squares. Write the formula for the function $f(x)$ that gives the volume of the box that can be formed by folding the resulting sheet of cardboard.

56. In 1990, the population of a small town was 2800. Each year since 1990, the population of the town has increased by 6%. Determine the formula for a function $P(t)$ that gives the population of the town t years after 1990.

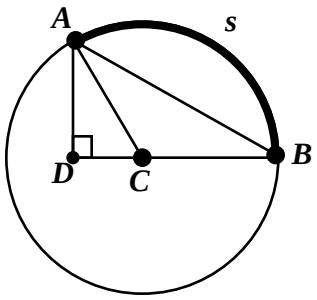
57. A ferris wheel is 36 meters in diameter and must be boarded from a platform that is 2 meters above he ground. The six o'clock position on the ferris wheel is level with the loading platform. The wheel completes one full revolution every 4 minutes. At $t = 0$ you are at the six o'clock position. Find a formula for the function $h(t)$ that gives your height above the ground after t minutes on the ferris wheel.

58. Solve the triangle shown at the right. Round your answers to the nearest hundredth.



59. Chris measures the angle of elevation to the top of a building as $38°$. When Chris moves 100 ft closer to the base of the building, the angle of elevation to the top of the building is $47°$. How tall is the building? Neglect Chris' height and round your answer to the nearest tenth of a foot.

60. In the figure at the right, C is the center of the circle, which has a radius of 6. The length of the arc s is 18. Determine the lengths of the line segments AD, DC, and AB, rounded to the nearest thousandth.



Answers

1. $\dfrac{1}{2\left(\sqrt{x+5}+\sqrt{x}\right)}$
2. $x\sqrt[15]{x^{11}}$
3. $\dfrac{x^2+2x+4}{x(3x-4)}$
4. $\dfrac{-x^2-9x+17}{(x-4)^2(x+1)}$
5. $y=-\dfrac{3}{5}x+\dfrac{23}{5}$
6. $\dfrac{-6\sqrt{2}+6}{7\pi}$

7. (a) $y = -\frac{2}{3}x + 8$ (b) For every two fewer coats he makes, he can make three additional shirts. (c) If the makes no shirts, he can make eight coats.

8. (a) $\frac{\sqrt{3}}{2}$ (b) –1 (c) $\frac{5\pi}{6}$ (d) 0

9. (a) $\frac{\sec x}{\cos x} = \frac{\sec x}{\frac{1}{\sec x}} = \sec^2 x = 1 + \tan^2 x$

(b) $\sin t(\csc t - \sin t) = \sin t\ \csc t - \sin t\ \sin t = \sin t\ \frac{1}{\sin t} - \sin^2 t = 1 - \sin^2 t = \cos^2 t$

(c) $\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2} = \cos x(0) - \sin x(1) = -\sin x$

(d) $\sin^2 x \cos^2 x = (\sin x \cos x)^2 = \frac{1}{4}(2\sin x \cos x)^2 = \frac{1}{4}(\sin 2x)^2 = \frac{1}{4}\left(\frac{1 - \cos 4x}{2}\right) = \frac{1}{8}(1 - \cos 4x)$

10. $\frac{2x}{x^2 + 1}$ 11. (a) r (b) (4,7) 12. (a) 1 (b) –1 13. $\frac{-10}{(2x + 2h - 3)(2x - 3)}$

14. (a) odd (b) symmetric about the origin 15. (a) $\frac{3}{9x^2 - 30x + 27}$ (b) $\frac{-5x^2 - 1}{x^2 + 2}$

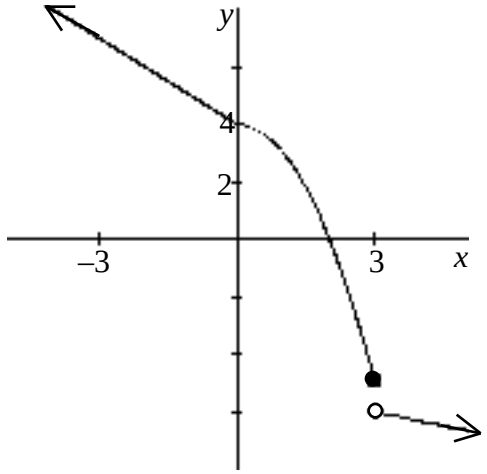
16. $f(x) = \sqrt[3]{x}$, $g(x) = x^2 + 9$ 17. $f^{-1}(x) = \frac{-7x - 5}{2x - 3}$ 18. $g^{-1}(x) = \arcsin\left(\frac{1}{x}\right)$

19. (a) $(-\infty, \infty)$ (b) $(-\infty, \infty)$ (c) $(-\infty, -4) \cup (-4, 2) \cup (2, \infty)$ (d) $\left(-\infty, \frac{2}{7}\right]$ (e) $(-\infty, \infty)$

(f) $\left(-\frac{5}{2}, \infty\right)$ (g) $(-\infty, \infty)$ (h) $\{x \mid x \quad , x \neq \frac{\pi}{2} + m\pi$, where m is an integer$\}$ (i) $\left[-\frac{1}{3}, \frac{7}{9}\right]$

(j) $(-\infty, \infty)$ (k) $(-\infty, \infty)$ (l) $(-3, 0) \cup (0, \infty)$ (m) $(-\infty, 0) \cup (0, \infty)$

20. (a)



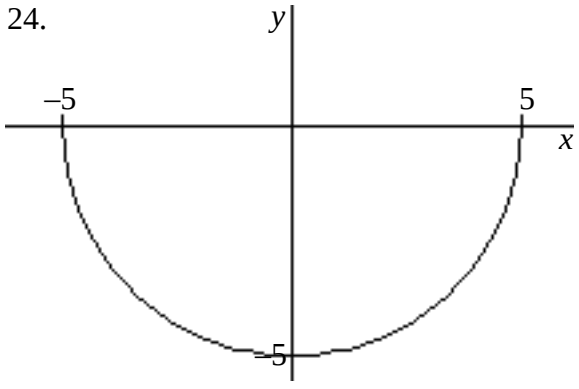
(b) Yes, by the Vertical Line Test.
(c) Yes, by the Horizontal Line Test.
(d) $f(3) = -5$ (e) $f^{-1}(-4) = 2\sqrt{2}$

21. (a) $\left(\frac{-1 \pm \sqrt{3}}{3}, 0\right)$ (b) (0,2)

22. (a) False (b) False (c) True (d) True

23. The graph is a circle with its center at $\left(\frac{3}{2}, -2\right)$ and with a radius of $\frac{\sqrt{15}}{2}$.

24.



25. $(-\infty, 4)$ 26. (a) $5^{-x} \to 0$ (b) $\log x \to -\infty$ (c) $\ln x \to \infty$ (d) $4^x \to 0$

27. (a) x^3 (b) x^4 (c) $y \to 0$ (d) $y \to \infty$ 28. $p(x) = -\frac{1}{6}(x+3)(x-2)^2$

29. (a) False (b) True (c) True (d) True (e) False (f) False (g) True (h) False

30. (a) π (b) 3 (c) $y = 5$ (d) 5 units up (e) $\frac{\pi}{4}$ units left (f) [2,8]

31. $-\frac{2}{23}$ 32. –4 33. –2, 5

34. $\frac{\frac{\ln 5.5}{\ln 5} + 3}{2} \approx 2.030$ 35. $\frac{\ln 48 - \ln 45}{\ln 1.038 - \ln 1.025} \approx 5.121$ years

36. $\left(-\frac{3}{2}, 0\right)$ 37. $\frac{403}{298}$ 38. 4

39. $\pi, \frac{\pi}{3}, \frac{5\pi}{3}$ 40. 0.560, 5.723 41. 1.249, 4.391, $\frac{3\pi}{4}, \frac{7\pi}{4}$

42. $\frac{3\pi}{4} + m\pi$, where m is an integer 43. $\left(-\infty, \frac{7}{5}\right)$

44. $\left(-\infty, -\frac{2}{3}\right) \cup (4, \infty)$ 45. $(-\infty, -3] \cup [5, \infty)$ 46. $(-\sqrt{7}, 0) \cup \left(0, \frac{5}{2}\right) \cup (\sqrt{7}, \infty)$

47. $(-\infty, -5) \cup \left(\frac{1}{7}, 4\right)$ 48. $(-\infty, -3)$ 49. $\left(-\infty, \frac{\ln 4}{\ln 7}\right]$

50. (2.5,502.5) 51. $\left[-4, -\frac{5}{3}\right)$ 52. $w = \frac{900}{v^2}$

53. $f(x) = \frac{x}{20} + \frac{30 - x}{6}$ 54. $f(x) = \sqrt{64 - x^2}$ 55. $f(x) = 4x^3 - 36x^2 + 80x$

56. $P(t) = 2800(1.06)^t$ 57. $h(t) = -18\cos\left(\frac{\pi}{2}t\right) + 20$

58. $b \approx 13.77$ ft, $A \approx 32.96°$, $C \approx 20.04°$

59. 287.8 ft

60. $AD \approx 0.847$, $DC \approx 5.940$, $AB \approx 11.970$