EXTRA PRACTICE ON APPLICATIONS OF LINEAR EQUATIONS

Here are a couple of reminders you may need for this section: perimeter is the distance around the outside of the figure (add the lengths of all sides to find perimeter), and the sum of the measures of the angles (not sides) in a triangle is 180 degrees.

For each of the following, (a) define the variable expression(s), (b) write an equation, (c) solve the equation and give all answers to the problem, and (d) check your answer. Try checking in the original written problem (the actual words) instead of checking in the equation you created. This is good practice because it is possible to make a mistake in creating the equation.

Example 1: If the sum of seven and a number is multiplied by four, the result is 76. Find the number.

(a) Let \( n \) = the number.

(b) The equation is \( 4(7 + n) = 76 \).

(c) To solve the equation, distribute the 4: \( 28 + 4n = 76 \). Then subtract 28 from both sides of the equation: \( 28 + 4n - 28 = 76 - 28 \), which becomes \( 4n = 48 \).

\[
\frac{4n}{4} = \frac{48}{4},
\]

which becomes \( n = 12 \). The number is 12.

(d) Checking with the words, first find the sum of 7 and 12 (that is, \( 7 + 12 = 19 \)). Then multiply the result by 4: \( 19 \cdot 4 = 76 \), which is the desired result so it checks.

Example 2: Six less than triple a number is 45. Find the number.

(a) Let \( n \) = the number.

(b) The equation is \( 3n - 6 = 45 \). [Note that whatever follows "less than" or "subtracted from" is always written before the subtraction symbol, and whatever appears before "less than" or "subtracted from" is written after the subtraction symbol.]

(c) To solve the equation, add 6 to both sides of the equation: \( 3n - 6 + 6 = 45 + 6 \), which becomes \( 3n = 51 \).

\[
\frac{3n}{3} = \frac{51}{3},
\]

which becomes \( n = 17 \). The number is 17.

(d) Checking with the words, first triple the number 17 (that is, \( 3 \cdot 17 = 51 \)). Then find 6 less than your result (that is, \( 51 - 6 = 45 \)). Since 45 is the desired result, the answer is correct.
Example 3: The perimeter of a rectangle is 170 inches. The length is eight inches less than double the width. Find the dimensions of the rectangle.

(a) Let \( x = \) the width of the rectangle in inches.  
    Then \( 2x - 8 = \) the length of the rectangle in inches.

(b) The formula for perimeter \( P \) of a rectangle is \( 2L + 2W = P \), where \( L \) is the length and \( W \) is the width. To create the equation, use either the formula above:  
    \( 2(2x - 8) + 2(x) = 170 \),  
    or add all four (not just two) sides of the rectangle and set this equal to the perimeter:  
    \( x + (2x - 8) + x + (2x - 8) = 170 \).  
    When simplified, both of the above equations become \( 6x - 16 = 170 \).

(c) To solve the equation, add 16 to both sides of the equation:  
    \( 6x - 16 + 16 = 170 + 16 \),  
    which becomes \( 6x = 186 \).  
    Then divide both sides by 6:  
    \[ \frac{6x}{6} = \frac{186}{6}, \]  
    which becomes \( x = 31 \).

From part (a) above, \( x \) is the width in inches, so the width is 31 inches.  
The length is \( 2x - 8 = 2(31) - 8 = 62 - 8 = 54 \), so the length is 54 inches.

(d) Check by either using the formula for perimeter of a rectangle:  
    \( 2(54 \text{ inches}) + 2(31 \text{ inches}) = 108 \text{ inches} + 62 \text{ inches} = 170 \text{ inches} \),  
    or by adding the four sides:  
    \( 31 \text{ inches} + 54 \text{ inches} + 31 \text{ inches} + 54 \text{ inches} = 170 \text{ inches} \), the desired result.

Example 4: A triangle has a perimeter of 33 meters. The second side of the triangle is four meters longer than the first side. The third side is 3 meters shorter than twice the first side. Find the length of each side of the triangle.

(a) Let \( x = \) the length of the first side of the triangle in meters.  
    Then \( x + 4 = \) the length of the second side of the triangle in meters.  
    Then \( 2x - 3 = \) the length of the third side of the triangle in meters.

(b) Add all three sides of the triangle and set this equal to the perimeter:  
    \( x + (x + 4) + (2x - 3) = 33 \).  
    When simplified, the equation becomes \( 4x + 1 = 33 \).

(c) To solve the equation, subtract 1 from both sides of the equation:  
    \( 4x + 1 - 1 = 33 - 1 \),  
    which becomes \( 4x = 32 \).  
    Then divide both sides by 4:  
    \[ \frac{4x}{4} = \frac{32}{4}, \]  
    which becomes \( x = 8 \).

From part (a) above, the length of the first side is \( x = 8 \) meters.  
The length of the second side is \( x + 4 = 8 + 4 = 12 \) meters.  
The length of the third side is \( 2x - 3 = 2(8) - 3 = 16 - 3 = 13 \) meters.

(d) Check by adding the three sides:  
    \( 8 \text{ meters} + 12 \text{ meters} + 13 \text{ meters} = 33 \text{ meters} \), the desired result.
Example 5: The largest angle of a triangle is 12 degrees more than the smallest angle. The third angle is 3 degrees less than the largest angle. Find the measure of each angle.

(a) Let \( x \) = the measure of the smallest angle of the triangle in degrees.
Then \( x + 12 \) = the measure of the largest angle of the triangle in degrees.
Then \( (x + 12) - 3 = x + 9 \) = the measure of the third angle of the triangle in degrees.

(b) The sum of the angles in a triangle is 180 degrees, so add the angles and set = 180°:
\[ x + (x + 12) + (x + 9) = 180. \]
When simplified, the equation becomes \( 3x + 21 = 180 \).

(c) To solve the equation, subtract 21 from both sides of the equation:
\[ 3x + 21 - 21 = 180 - 21, \]
which becomes \( 3x = 159 \).
Then divide both sides by 3:
\[ \frac{3x}{3} = \frac{159}{3}, \]
which becomes \( x = 53 \).

From part (a) above, the measure of the smallest angle is \( x = 53 \) degrees.
The measure of the largest angle is \( x + 12 = 53 + 12 = 65 \) degrees.
The measure of the third angle is \( x + 9 = 53 + 9 = 62 \) degrees.

(d) Check by adding the measures of the three angles:
\[ 53 \text{ degrees} + 65 \text{ degrees} + 62 \text{ degrees} = 180 \text{ degrees}, \]
the desired result.

Example 6: The sum of two numbers is 42. Three times the smaller number plus twice the larger number is 97. Find the two numbers.

(a) Let \( x \) = one number. Since the sum of the two numbers is 42, the other number = 42 - \( x \).
[For example, if the first number was 10, the other number would be 42 - 10 = 32]. If the problem says that the sum of two numbers is something, you should be able to let \( x \) represent the first number and the sum - \( x \) represent the second number. In summary:
Let \( x \) = the smaller number
Then 42 - \( x \) = the larger number
It does not matter which you call the larger or smaller number except that you must use your choices appropriately in part (b) below:

(b) Use the second sentence to set up the equation:
\[ 3(\text{smaller}) + 2(\text{larger}) = 97 \]
Put the appropriate variables in place of smaller and larger:
\[ 3(x) + 2(42 - x) = 97. \]
This becomes \( 3x + 84 - 2x = 97 \).
Combine like terms to get \( x + 84 = 97 \).

(c) Subtract 84 from both sides of the equation:
\[ x + 84 - 84 = 97 - 84, \]
which becomes \( x = 13 \).
From part (a) above, the smaller number is \( x = 13 \).
The larger number is 42 - \( x = 42 - 13 = 29 \).

(d) Check your answers in both parts (sentences) of the problem:
\[ 13 + 29 = 42, \text{ and } 3(13) + 2(29) = 97. \]
Since both equations are true, the answers are correct.
Work the following problems. For each problem, (a) define the variable expression(s), (b) write an equation, (c) solve the equation and give all answers to the problem, and (d) check your answer.

1. If five times a number is increased by 3, the result is 38. Find the number.

2. If twice a number is decreased by four, the result is eighteen. Find the number.

3. If the sum of a number and eight is multiplied by six, the result is 126. Find the number.

4. The perimeter of a rectangle is 82 feet. The length is 7 feet longer than the width. Find the dimensions of the rectangle.

5. The perimeter of a rectangle is 64 centimeters. The length is 4 centimeters shorter than triple the width. Find the dimensions of the rectangle.

6. A triangle has a perimeter of 63 inches. The second side of the triangle is six inches shorter than the first side. The third side is 9 inches longer than double the first side. Find the length of each side of the triangle.

7. The perimeter of a triangle is 52 meters. The second side of the triangle is five times the first side. The third side is 2 meters shorter than three times the first side. Find the length of each side of the triangle.

8. A 137-ft long rope is cut into three pieces. The length of the second piece is twice the length of the first piece. The third piece is 9 ft longer than the first piece. Find the length of each piece of rope.

9. The largest angle of a triangle is 21 degrees more than the middle-sized angle. The smallest angle is 27 degrees less than the largest angle. Find the measure of each angle.

10. The sum of two numbers is 22. The larger number plus three times the smaller number is 38. Find the numbers.

11. The sum of two numbers is 55. Their difference is 19. Find the numbers.

12. The sum of three consecutive odd integers is -87. Find the integers.
ANSWERS:

1. a) Let \( n \) = the number
   
b) \( 5n + 3 = 38 \)
   
c) The number is 7.
   
d) \( 5(7) + 3 = 35 + 3 = 38 \) ✓

2. a) Let \( n \) = the number
   
b) \( 2n - 4 = 18 \)
   
c) The number is 11.
   
d) \( 2(11) - 4 = 22 - 4 = 18 \) ✓

3. a) Let \( n \) = the number
   
b) \( 6(n + 8) = 126 \)
   
c) The number is 13.
   
d) \( 6(13 + 8) = 6(21) = 126 \) ✓

4. a) Let \( x \) = the width of the rectangle in ft
   Then \( x + 7 \) = the length of the rectangle in ft
   
b) \( 2(x + 7) + 2(x) = 82 \), or
   \( x + (x + 7) + x + (x + 7) = 82 \), or
   \( 4x + 14 = 82 \)
   
c) The width is 17 ft.
   The length is 24 ft.
   
d) \( 2(24 \text{ ft}) + 2(17 \text{ ft}) = 82 \text{ ft} \) ✓

5. a) Let \( x \) = the width of the rectangle in cm
   Then \( 3x - 4 \) = the length of the rectangle in cm
   
b) \( 2(3x - 4) + 2(x) = 64 \), or
   \( x + (3x - 4) + x + (3x - 4) = 64 \), or
   \( 8x - 8 = 64 \)
   
c) The width is 9 cm.
   The length is 23 cm.
   
d) \( 2(23 \text{ cm}) + 2(9 \text{ cm}) = 64 \text{ cm} \) ✓
EXTRA PRACTICE ON APPLICATIONS OF LINEAR EQUATIONS

ANSWERS:

6. a) Let $x =$ the length of the first side in inches
Then $x - 6 =$ the length of the second side in inches
Then $2x + 9 =$ the length of the third side in inches

b) $x + (x - 6) + (2x + 9) = 63$, or
$4x + 3 = 63$

c) The sides are 15 inches, 9 inches, and 39 inches.

d) 15 inches + 9 inches + 39 inches = 63 inches $\sqrt{\quad}$

7. a) Let $x =$ the length of the first side in meters
Then $5x =$ the length of the second side in meters
Then $3x - 2 =$ the length of the third side in meters

b) $x + 5x + (3x - 2) = 52$, or
$9x - 2 = 52$

c) The sides are 6 meters, 30 meters, and 16 meters.

d) 6 m + 30 m + 16 m = 52 m $\sqrt{\quad}$

8. a) Let $x =$ the length of the first piece in ft
Then $2x =$ the length of the second piece in ft
Then $x + 9 =$ the length of the third piece in ft

b) $x + 2x + (x + 9) = 137$, or
$4x + 9 = 137$

c) The pieces are 32 ft, 64 ft, and 41 ft.

d) 32 ft + 64 ft + 41 ft = 137 ft $\sqrt{\quad}$

9. a) Let $x =$ the measure of the middle-sized angle in degrees
Then $x + 21 =$ the measure of the largest angle in degrees
And largest - 27 = $(x + 21) - 27 = x - 6 =$ the measure of the smallest angle in degrees

b) $x + (x + 21) + (x - 6) = 180$, or
$3x + 15 = 180$

c) The measures of the angles are 55 degrees, 76 degrees, and 49 degrees.

d) 55 degrees + 76 degrees + 49 degrees = 180 degrees $\sqrt{\quad}$
10. a) Let \( x \) = the larger number  
   Then \( 22 - x \) = the smaller number  

b) Larger + 3(smalller) = 38, or  
   \( x + 3(22 - x) = 38 \), or \( x + 66 - 3x = 38 \), or  
   \(-2x + 66 = 38\)  

c) The numbers are 14 and 8.  

d) \( 14 + 8 = 22 \)  
   \( 14 + 3(8) = 38 \)  

11. a) Let \( x \) = one number  
   Then \( 55 - x \) = the other number  

b) To find the difference between two numbers, subtract them:  
   One number - the other number = 19, or  
   \( x - (55 - x) = 19 \), or \( x - 55 + x = 19 \), or  
   \( 2x - 55 = 19\)  

c) The numbers are 37 and 18.  

d) \( 37 + 18 = 55 \)  
   \( 37 - 18 = 19 \)  

12. a) Let \( x \) = the first odd integer  
   Then \( x + 2 \) = the second odd integer  
   And \( x + 4 \) = the third odd integer  
   [Note that since odd numbers are two apart, you must add 2 to get to the next odd  
   integer. For example, if \( x = 15 \), the next odd integer is \( x + 2 = 17 \). Since even  
   numbers are also two apart, the same setup applies to consecutive even integers.]  

b) First odd integer + second odd integer + third odd integer = -87, or  
   \( x + (x + 2) + (x + 4) = -87 \), or  
   \( 3x + 6 = -87\)  

c) \( x = -31 \)  
   \( x + 2 = -31 + 2 = -29 \)  
   \( x + 4 = -31 + 4 = -27 \)  
   The consecutive odd integers are -31, -29, and -27.  

d) \(-31 + -29 + -27 = -87 \)  