Laboratory: Archimedes' Principle

Introduction:

Eureka!! In this lab your goal is to perform some experiments to understand the source of Archimedes' excitement when he discovered that the buoyant force on an object in a liquid is equal to the weight of the liquid that the object displaces.

Archimedes made his discovery without the benefit of Newton's insights. He was arguably the premier scientist of antiquity. Since you have Newton's achievements as part of your intellectual heritage (Phy211) we'll use the concepts of force and equilibrium to measure the buoyant force acting on a (partially or totally) submerged object.

Here is the idea behind the experiment. Suppose we hang a small mass from a force sensor, arranged in the configuration shown in the figure below. The bob in the liquid may be partially or totally submerged.

H_2O

Force Sensor

mass
graduated
cylinder

According to Archimedes, the upward buoyant force (**F_B**), which the liquid exerts on the object, is equal to the weight of the fluid displaced or:

$$F_B = m_{fluid}g = (\rho_{fluid}V)g \quad (1).$$

It should be noted, the buoyant force produced by the displacement of fluid in the cylinder will result in a reaction force of equal but opposite magnitude exerted by the bob on the liquid (according to Newton's 3rd Law). We will use this action-reaction relation to measure the buoyant "reaction" force with a standard digital scale. While the mass is hanging, the force sensor will measure the tension in the string (F_{T1}), which should be equal to the weight of the suspended mass. Applying Newton's 2nd Law to the hanging mass:

$$F_{Net} = F_{T1} - mg = 0$$

Or

$$F_{T1} = mg \quad (2).$$

When the mass is inserted into the liquid, the force sensor will measure the new tension in the string (F_{T2}) which should be equal to the weight of the mass minus the buoyant force, **F_B**. Thus, when we look at the forces acting on the hanging mass:

$$F_{Net} = F_{T2} - mg + F_B = 0$$

or

$$F_B = mg - F_{T2} = F_{T1} - F_{T2} = \Delta F_T \quad (3).$$

Combining equations (1) and (3), the relationship between the buoyant force, the normal force (from the scale) and weight of the cylinder and liquid is given by:

$$\mathbf{F_B = F_{T1} - F_{T2} = (\rho_{fluid}V)g}$$

or

$$\mathbf{F_B = F_{T1} - F_{T2} = (\rho_{fluid} \cdot g)V} \quad (4).$$

finger
cup

Scale

liquid

Thus the buoyant force is linearly related to the displaced volume of the fluid, V, and the slope of a graph of F_B vs V should yield $\rho_{fluid} \cdot g$.

The Experiment:

Part 1: Qualitative Observations

1) Obtain a digital scale and zero it.

2) Obtain a Styrofoam cup and fill it halfway with water.

3) Place the cup on a digital scale. Observe the scale reading. Record it:

____

4) Insert your finger partially into the water. Observe the scale reading and record it:

____

5) Insert finger further into the water. Observe the scale reading. Record it:

____

6) Insert finger even further into the water. Observe the scale reading. Record it:

____

Question: How does the scale reading change as your finger is submerged into the water?

Part 2. The density of water

1. Obtain a set of cylinder samples from the instructor.

2. Measure the dimensions of each cylindrical sample and record the values in the table below.

3. Calculate the volume of each sample and record the values in the data table.

4. Obtain a 500-1000 mL beaker and fill it approximately half full with tap water, The beaker should have enough fluid to completely submerge your tallest sample.

3. Connect a dual-range force sensor to the LabPro interface and set it to ±10 N.

4. Start the LoggerPro software and calibrate the force sensor using a 100 or 200 g mass.

5. Set up an experiment to measure the tension force in a string suspended from the force sensor.

   a. Set up a ring stand and attach the force sensor to a support beam
   b. Attach a length of string to the force sensor then hang a cylindrical sample by the string.
   c. Collect ~2 s of force sensor readings using LoggerPro then highlight the data using the mouse cursor.
   d. Use the "Statistics" tool to obtain the average force sensor value. Record the average force sensor value in the data table.
   e. Carefully lower the cylinder into the water-filled beaker until it is complete submerged in the fluid. It is okay to go a little, but not too far, below the surface of the water (this will help to avoid surface tension measurement artifacts).
   f. Collect ~2 s of force sensor readings and highlight the data using the mouse cursor.
   g. Obtain and record the average force sensor reading in the data table.
   h. Repeat steps (b)–(g) for each of your cylinder samples.

				(out of water)	***(submerged)***	
Sample ID	**Diameter (m)**	**Length (m)**	**Volume (m^3)**	**Force Reading, F_{T1} =mg (in N)**	**Force Reading, F_{T2} = mg-F_B (N)**	**ΔF_T (= F_B) {The buoyant force}**

5. Using Graphical Analysis, create a graph of buoyant force (F_B) vs. the volume of the water displaced. Use the "Linear Fit" function to calculate the "Best-Fit" line for the data. Record the slope value for the linear fit.

Slope = _______________ (don't forget the units…)

6. Use the slope of the graph and Equation (4) above to determine the density of the water (ρ_{H2O}). Be sure to express your answer in SI units. To quantify the accuracy of your

density value, calculate the % Error. The accepted density of pure water at 20°C is 998 kg/m^3.

ρ_{H2O} = ________________ **% Error =** _____________________

Part 3. The density of an irregular mass

In this last section, you will use your measured density of water to determine the density of an "unknown" hanging mass.

1. Obtain an "unknown" mass sample from the instructor and suspend it by string from the force sensor.
2. Collect ~2 s of force sensor readings and use the "Statistics" tool to obtain the average force sensor value. Record the average force sensor value:

F_{T1} = mg = ______ N

3. Carefully lower the mass into the water-filled beaker until it is complete submerged.
4. Collect ~2 s of force sensor readings and obtain the average force sensor reading. Record this value:

F_{T2} = mg – F_B = ______ N

5. Calculate the buoyant force exerted on the submerged mass:

F_B = F_{T1} - F_{T2} = ______ N

6. Using this data, calculate the density of the hanging mass.
   Hints:
   - *The weight of the object is given by:* **W = mg = ρgV**
   - *The volume of the mass is the same as the volume of displaced water when it is submerged*
7. Record the density of the mass sample. Compare this value to the accepted density of lead (use the CRC Handbook to obtain this value).

$\rho_{unknown}$= _____________________

$\rho_{accepted}$= _____________________ **% Error =**_____________________