PHYS 1401
General Physics I
EXPERIMENT 15

STANDING WAVES ON A STRETCHED STRING

I. INTRODUCTION

The objective of this experiment is to study the standing waves on a stretched string. One end of this sting is connected to a fixed-frequency oscillator and the other end is fixed. The student will observe the first four normal modes of vibration of the string and measure their wavelengths. From this measurement, the frequency will be calculated and compared with that of the oscillator.

II. APPARATUS

Mechanical oscillator ($f = 120\,\text{Hz}$), string, pulleys, clamps and assortment of masses.

III. THEORY

A transverse disturbance on a stretched string moves along the string with a speed

$$v = \sqrt{\frac{T}{\mu}}. \tag{1}$$

where T is the tension in the string and μ is the linear mass density (mass per unit length). Also the speed is the product of the frequency and the wavelength

$$v = \lambda f. \tag{2}$$

A standing wave is set up on the string as a result of the interference of the incident wave (generated by the oscillator) and the wave reflected from the other end of the string. If the incident wave is moving in the positive x-direction, its wave function can be written as

$$y_{\text{inc}} = A\cos(kx - \omega t) \tag{3}$$

where A is the wave amplitude, k is the wavenumber and ω is the angular frequency. The wave reflected from the other end of the string is moving in the negative x-direction and its wave function can be written as

$$y_{\text{ref}} = A\cos(kx + \omega t). \tag{4}$$

The resultant wave is the superposition of these two waves and its wave function is the sum of the two wave functions

$$\begin{aligned} y &= y_{\text{inc}} + y_{\text{ref}} && (5)\\ &= A\cos(kx - \omega t) + A\cos(kx + \omega t) && (6)\\ &= 2A\sin(kx)\cos(\omega t). && (7) \end{aligned}$$

The last expression shows that the various segments of the string now oscillate in simple harmonic motion $(\cos\omega t)$ with an amplitude which varies sinusoidally along the length of the string $(2A\sin(kx))$. For certain values of frequencies (and wavelengths), standing wave patterns are produced on the string. For our set up, since the string is fixed at both ends, we would like to set the amplitude to be zero at both ends. Setting $\sin(kL) = 0$ gives a relationship between the wavelengths of the various modes and the length of the string. This relation is $\lambda_n = (2L/n)$ where n is the mode number. Also setting $\sin(kx) = 0$ gives the location of the nodes (zero amplitude points) along the string. These nodes are located $\lambda/2$ apart.

IV. EXPERIMENTAL PROCEDURE

1. Cut a $2\,\mathrm{m}$ long piece of the string and measure its mass on the digital scale. Calculate the mass of the string per unit length. This is μ in equation (1).

2. Clamp the oscillator onto one side of the lab table. Tie one end of the string to the oscillator and run the other end over a pulley clamped on the other side of the table.

3. To observe the various standing wave modes, the tension in the string must have a value consistent with the resonance condition.

4. Hang a mass of $1.250\,\mathrm{kg}$ from the free end (remember that the hanger has a mass of $0.050\,\mathrm{kg}$). Plug the oscillator in the electrical outlet.

5. Place a wooden wedge on the table near the oscillator such that the thin end is pushing the string and start moving it slowly towards the other end of the string. Continue moving the wedge until you encounter a node in the standing wave pattern. This is a point of zero amplitude. With the wedge located at this position, the standing wave on the string is observable. Adjust the position of the wedge until the standing wave is stable and the antinode has its maximum amplitude. Measure the distance between the oscillator (considered a node) and the wedge. This is one half of a wavelength $(\lambda_1/2)$. If the node does not seem to be at the oscillator but very close to it, measure the half wavelength from that point.

6. Reduce the hanging mass to $M = 1.050\,\mathrm{kg}$ and repeat the process above. Keep moving the wedge until it is one full wavelength from the oscillator and measure the wavelength, λ_2.

7. Reduce the hanging mass to $M = 0.550\,\mathrm{kg}$ and move the wedge until it is $(3/2)\lambda_3$ from the oscillator. Measure this distance.

8. Now you would like to set up the standing wave pattern which has two wavelengths. Choose a hanging mass and repeat the process above until you succeed at generating such a pattern. Measure the distance between the oscillator and the wedge.

IV. ANALYSIS

1. For each mode, calculate the tension in the string, $T = Mg$, and record it in the data table.

2. For each mode, calculate the speed of the original incident wave from equation (1) and enter it in the data table.

3. For each mode, calculate the wavelength.

4. For each mode, calculate the frequency from equation (2).

5. Calculate the percent difference between the calculated frequency and the oscillator frequency

$$\%\,\text{diff.} = \frac{|f - 120|}{120} \times 100. \tag{8}$$

6. Draw the observed four normal modes. Label the nodes and antinodes.

7. Use the resonance condition and find a relationship between the length of the string (between the oscillator and the wedge) and the wavelength for each mode.

8. Write a conclusion. Comment on the accuracy of this experiment. What are the two most important sources of error in this experiment?

Table (1): Standing Waves			
Hanging Mass M (kg)	Tension in the String $T = Mg$ (N)	Number of Antinodes n	Wave Speed v (m/s)
		1	
		2	
		3	
		4	

Table (2): Standing Waves			
Measured Distance: Oscillator to Wedge (m)	Wavelength λ (m)	Frequency $f_n = \frac{v}{\lambda_n}$	% Difference