PHYS 2425
Engineering Physics I

EXPERIMENT 11
STANDING WAVES ON A STRETCHED STRING

I. INTRODUCTION

The objective of this experiment is to study the standing waves on a stretched string. One end of this sting is connected to a fixed-frequency oscillator and the other end is fixed. The student will observe the first four normal modes of vibration of the string and measure their wavelengths. From this measurement, the frequency will be calculated and compared with that of the oscillator.

II. THEORY

A transverse disturbance on a stretched sting moves along the string with a speed

$$v = \sqrt{\frac{T}{\mu}}. \tag{1}$$

where T is the tension in the string and μ is the linear mass density (mass per unit length). Also the speed is the product of the frequency and the wavelength

$$v = \lambda f. \tag{2}$$

A standing wave is set up on the string as a result of the interference of the incident wave (generated by the oscillator) and the wave reflected from the other end of the string. If the incident wave is moving in the positive x-direction, its wave function can be written as

$$y_{\text{inc}} = A\cos(kx - \omega t) \tag{3}$$

where A is the wave amplitude, k is the wavenumber and ω is the angular frequency. The wave reflected from the other end of the string is moving in the negative x-direction and its wave function can be written as

$$y_{\text{ref}} = A\cos(kx + \omega t). \tag{4}$$

The resultant wave is the superposition of these two waves and its wave function is the sum of the two wave functions

$$\begin{aligned} y &= y_{\text{inc}} + y_{\text{ref}} \\ &= A\cos(kx - \omega t) + A\cos(kx + \omega t) \\ &= (2A\sin(kx))\cos(\omega t). \end{aligned} \tag{5}$$

The last expression shows that the various segments of the string now oscillate in simple harmonic motion $(\cos\omega t)$ with an amplitude which varies sinusoidally along the length of the string $(2A\sin(kx))$. For certain values of frequencies (and wavelengths), standing wave patterns are produced on the string. For our set up, since the string is fixed at both ends, we would like to set the amplitude to be zero at both ends. Setting $\sin(kL) = 0$ gives a relationship between the wavelengths of the various modes and the length of the string. This relation is $\lambda_n = (2L/n)$ where n is the mode number. Also setting $\sin(kx) = 0$ gives the location of the nodes (zero amplitude points) along the string. These nodes are located $\lambda/2$ apart.

III. APPARATUS

Mechanical oscillator ($f = 120\,\mathrm{Hz}$), string, pulleys, clamps and assortment of masses.

IV. EXPERIMENTAL PROCEDURE

1. Cut a $2\,\mathrm{m}$ long piece of the string and measure its mass on the digital scale. Calculate the mass of the string per unit length.

2. Clamp the oscillator onto one side of the lab table. Tie one end of the string to the oscillator and run the other end over a pulley clamped on the other side of the table.

3. To observe the various standing wave modes, the tension in the string must be a value consistent with the resonance condition. So you are looking for the correct amount of hanging mass which will allow the desired mode to be excited. You will attempt to observe the first harmonic, $n = 1$, first and find the correct tension for it by trial and error. The tension in the string is the weight of the hanging mass. Start by hanging a mass $M = 0.500\,\mathrm{kg}$ on the free end of the string.

4. If you do not observe any standing waves, hang another $0.500\,\mathrm{kg}$. Then start adding $0.100\,\mathrm{kg}$ masses. When you get to a total hanging mass $M = 1.500\,\mathrm{kg}$, and if you have not observed any standing waves yet, continue adding mass by an increment of $10\,\mathrm{g} = 0.01\,\mathrm{kg}$. You will know when you are close to exciting the desired standing wave. Now add smaller increments of mass until there is exactly 1 lobe along the entire length of the string.

5. Measure the distance between each two consecutive nodes. This distance is $(\lambda/2)$. Record these distances in the data table and find their average. This will be taken as $\lambda_1/2$. Calculate λ_1 the wavelength for this mode and enter it in the data table.

6. Reduce the amount of hanging mass to allow the second harmonic, $n = 2$, to be excited. Repeat what you did above to get exactly 2 lobes along the entire length of the string.

7. Measure the distances between each two consecutive nodes, record them in the data table and find their avergae. Call this average $\lambda_2/2$. Calculate λ_2 and enter it in the data table.

8. Repeat the above steps to excite the third and fourth harmonic modes and measure their wavelengths.

IV. ANALYSIS

1. For each mode, calculate the tension in the string, $T = Mg$, and record it in the data table.

2. For each mode, calculate the speed of the original incident wave from equation (1) and enter it in the data table.

3. For each mode, calculate the frequency from equation (2).

4. Calculate the percent difference between the calculated frequency and the oscillator frequency

$$\% \text{ Percent diff.} = \frac{|f - 120|}{120} \times 100. \tag{6}$$

5. Draw the observed four normal modes. Label the nodes and antinodes.

6. Use the resonance condition and find a relationship between the length of the string and the wavelength for each mode.

7. Write a conclusion. Comment on the accuracy of this experiment. What are the two most important sources of error in this experiment?

Table (1): Standing Waves			
Hanging Mass M (kg)	Tension in the String $T = Mg$ (N)	Number of Antinodes n	Wave Speed v (m/s)
		1	
		2	
		3	
		4	
Table (2): Standing Waves			
Measured Distance: Oscillator to Wedge (m)	Wavelength λ (m)	Frequency $f_n = \frac{v}{\lambda_n}$	% Difference