

PHYS 2425
Engineering Physics I

EXPERIMENT 4
WORK-ENERGY

I. INTRODUCTION

The objective of this experiment is to test the work-energy theorem which states that the net work done on an object is equal to the change in the kinetic energy of that object, that is

$$W_{\text{net}} = \Delta K. \quad (1)$$

The student will measure the velocity of an air track glider of mass M at two positions and calculate the change in the kinetic energy of the glider between these two positions

$$\Delta K \equiv K_1 - K_0 = \frac{1}{2}Mv_1^2 - \frac{1}{2}Mv_0^2. \quad (2)$$

The net work done on the glider will be calculated from the net force acting on the glider and the distance it moved between the two positions.

II. THEORY

Work Done by a Constant Force

Consider an air track glider of mass M connected to a hanging mass m by a string which goes over a smooth pulley attached at the end of the air track. By applying Newton's second law to the glider and the hanging mass, it can be shown that the tension in the string is

$$T = \frac{mM}{m + M}g. \quad (3)$$

The work done by this tension on the glider is the product of the tension and the distance the glider moved

$$W_{\text{net}} = Th_0 = \frac{mMgh_0}{m + M} \quad (4)$$

The distance the glider moved, h_0 is the distance from the bottom of the hanging mass to the floor.

Work Done by a Variable Force

If the force, F , acting on the object is variable (not constant), then the work done by this force on the object as it moves from point x_0 to point x_1 is

$$W = \int_{x_0}^{x_1} F(x)dx \quad (5)$$

For the purposes of this experiment, the net force acting on the glider is the tension in the string. If we replace the hanging mass at the end of the string by a chain of

mass m and length L , then as the glider is pulled along the air track by the falling chain, the chain will start to pile up on the floor and the net force acting on the system will decrease. This will give us a tension in the string which varies with distance. From Newton's second law, it is straightforward to show that the tension in the string is given by

$$T = \frac{\lambda(h_0 - x)g}{1 + \frac{\lambda(h_0 - x)}{M}}. \quad (6)$$

The work done by the tension on the glider is

$$\begin{aligned} W_{\text{net}} &= \int_0^{h_0} T(x)dx \\ &= \frac{gM^2}{\lambda} \left[\frac{\lambda h_0}{M} - \ln \left(\frac{\lambda h_0}{M} + 1 \right) \right] \end{aligned} \quad (7)$$

In equations (6) and (7), h_0 is the distance from the top of the hanging chain to the floor, λ is the mass per unit length of chain $\lambda = m_c/L$ (m_c is the mass of the chain) and M is the mass of the glider.

III. APPARATUS

Air track with accessory kit, assortment of masses, chain, photogate and computer.

IV. EXPERIMENTAL PROCEDURE

1. Level the air track.
2. Affix a small flag on the glider and set up the photogate to the right height for the flag to pass under it and still block the beam when it goes through it (see Figure (1)).

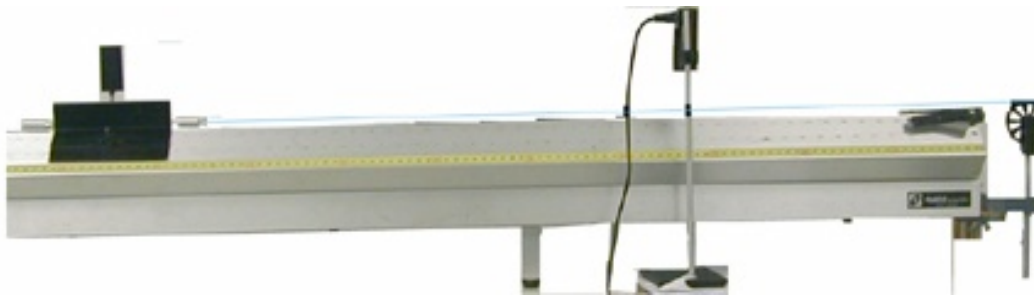


Figure 1: Air Track with Photogate

3. Measure the width of the flag using the following procedure:
 - (a) Record the position of the front of the glider when the beam is first blocked.

- (b) Move the glider slowly until the beam is unblocked and record the position of the front of the glider again.
 - (c) The width of the flag is the difference between these two numbers. This number will be entered in the computer momentarily.
4. Open the **Logger Pro** folder on the computer. Then open the file **One Gate Timing**.
 5. Go to **Experiment** on the menu bar at the top and drag down to sampling. Change the number in the length box to the value you obtained for the length of the flag (in meters).
 6. Tie a string (about 1.5 meters long) to the hanging mass (see data table) and tie the other end of it to the hook which fits in the top hole of the air track glider.
 7. Measure the hanging mass, m and the glider mass M using the digital balance and record the numbers in the data table. Don't forget to record the uncertainties.
 8. Ensure that the flag passes through the photogate just after the hanging mass hits the floor.
 9. Pull the glider back until the hanging mass is a few centimeters below the pulley and measure the distance from the floor to the bottom of the hanging mass. This distance is h_0 . You are ready to collect the data.
 10. Press **return** and wait for the red **Collect** button on the menu bar to change to **Stop**. Release the glider and then catch it **after it passes through the photogate**. Record the velocity of the glider in the appropriate place in the data table. This velocity is the velocity of the glider as it passed through the photogate. If there is data in the data table (from previous users), press return to erase this data before the stop button appears.
 11. Repeat the above step using the masses given in the data table. These masses are nominal values and you should measure these masses and record them with their uncertainties.
 12. You will collect 3 data points for the constant force case as indicated by the data table.
 13. For the variable force case, the chain will replace the hanging mass. Measure the length and the mass of the chain, calculate the mass per unit length (in kg/m) and record these values in the data table.
 14. Connect one end of the chain to the string and let the other end fall to the floor. Part of the chain will lie on the floor.

15. Pull the glider back until the top of the chain is a few centimeters below the pulley and the chain hangs straight down. Measure the distance from the floor to the top of the chain. This is h_0 . Note the position of the front of the glider at this point so you can start from this position for every run keeping the value of h_0 constant.
16. Position the photogate such that the flag passes through it just after the entire chain is on the floor (no part of it is hanging).
17. Start the timer as before and release the glider. Record the glider velocity.
18. Repeat the above step using the masses given in the data table. For each case record the velocity of the glider in the data table.
19. You are done with the experimental procedure. Leave your apparatus as it is and do some sample calculations to see how close is the experimental result to the theory before you leave the lab.

V. ANALYSIS

1. Draw the free body diagram of the glider and the hanging mass, write Newton's second law for each and derive eq. (3) for the tension.
2. Calculate the kinetic energy of the glider for each case and enter it in the data table.
3. Calculate the net work done on the glider using eq. (4) for the constant force cases (first 3 cases) and eq. (7) for the variable force cases (last 3 cases) and enter them in the data table.
4. Calculate the percent difference between W_{net} and ΔK and enter it in the data table.

$$\% \text{ difference} = \frac{W_{\text{net}} - K}{\left(\frac{W_{\text{net}} + K}{2}\right)} \times 100. \quad (8)$$

The work energy theorem states that these two quantities are equal.

5. Calculation of experimental error:
 - (a) The uncertainties δh_0 , δm , δM and δL_f should have been recorded as part of your data. L_f is the flag width. Calculate the relative uncertainty in the net work done and enter it in the data table,

$$\left|\frac{\delta W}{W}\right| = \left|\frac{\delta m}{m}\right| + \left|\frac{\delta M}{M}\right| + \left|\frac{\delta h_0}{h_0}\right| + \left|\frac{\delta(m+M)}{(m+M)}\right| \quad (9)$$

- (b) Calculate the relative uncertainty in the kinetic energy and enter it in the data table.

$$\left| \frac{\delta K}{K} \right| = \left| \frac{\delta M}{M} \right| + 2 \left| \frac{\delta v}{v} \right|. \quad (10)$$

You do not know δv . The computer found v by dividing the length of the flag, L_f by the time it took the flag to pass through the photogate. That is, $v = L_f/t$. The computer calls this t delta t. Then we have

$$\left| \frac{\delta v}{v} \right| = \left| \frac{\delta L_f}{L_f} \right| + \left| \frac{\delta t}{t} \right|. \quad (11)$$

The width of the flag was found by subtracting two measurements which you read off the air track scale. Hence $\delta L_f = 0.5 \text{ mm} = 0.05 \text{ cm}$. The computer will give you a time measurement for every velocity measurement. From this measurement, record the time it took the flag to pass through the photogate. Now we can calculate the relative uncertainty in the kinetic energy.

$$\left| \frac{\delta K}{K} \right| = \left| \frac{\delta M}{M} \right| + 2 \left| \frac{\delta L_f}{L_f} \right| + 2 \left| \frac{\delta t}{t} \right|. \quad (12)$$

6. Are any of the percent differences or the relative uncertainties greater than 10%?
7. Write a conclusion. In your conclusion comment on the success of this experiment. Are your results consistent with the work-energy theorem? Comment on any percent differences that are larger than 10%. What do you think are the two major sources of error in this experiment?
8. EXTRA CREDIT:
 - (a) For the variable force cases, derive the expression for the tension, eq. (6), and the expression for W_{net} , eq. (7).
 - (b) For the variable force cases, first derive an expression for $\frac{\delta W_{\text{net}}}{W_{\text{net}}}$ and then calculate numerical values for the 3 cases and record them in the data table.

Experiment (4) Data Table
$L_f \pm \delta L_f =$
$h_0 \pm \delta h_0 =$
$L_{ch} \pm \delta L_{ch} =$
$m_{ch} \pm \delta m_{ch} =$
$\lambda = \frac{m_{ch}}{L_{ch}} =$

Experiment (4) Data Table					
Hanging Mass $m \pm \delta m$ (kg)	Glider Mass $M \pm \delta M$ (kg)	Glider Velocity v (m/s)	Glider Kinetic Energy K (J)	Work Done on Glider W_{net} (J)	Time t (s)
Constant Force					
Variable Force					
Chain					
Chain					
Chain					

Experiment (4) Results Table		
% Difference Between K and W_{net}	$\frac{\delta K}{K}$ %	$\frac{\delta W_{net}}{W_{net}}$ %
Constant Force		
Variable Force		
Chain		
Chain		
Chain		