

**PHYS 2425**  
**Engineering Physics I**  
**EXPERIMENT 7**  
**ROTATIONAL DYNAMICS**

**I. INTRODUCTION**

The objective of this experiment is to measure the angular acceleration of a rotating disc and to test Newton's second law for rotation which states

$$\sum \tau = I\alpha \quad (1)$$

where  $\sum \tau$  is the net external torque,  $I$  is the moment of inertia (rotational inertia) of the disc and  $\alpha$  is the angular acceleration.

**II. APPARATUS**

PASCO rotational apparatus, steel ring, smart pulley, auxiliary pulley, assortment of masses, string and a computer.



Figure 1: PASCO Rotational Apparatus

**III. EXPERIMENTAL PROCEDURE**

1. Set up the apparatus as shown in figure (1). Use a rubber band to hold the smart pulley against the edge of the disc with as weak a force as possible. Insert the auxiliary pulley into the piece attached to the leg and tighten the screw or the clamp.

2. Use a string roughly 1 meter long, thread one end of the string through the small hole at the top of the step pulley (which is at the center of the disc) and tie it up. Run the string over the auxiliary pulley and attach the hanging mass to its end. Adjust the height of the pulley such that the string runs horizontal.
3. Turn the computer on. Plug the smart pulley into input port Dig/Sonic1 of the LabPro. Double click on **Logger Pro** and then double click on **Rotational Apparatus**. This will open a "rotational velocity vs. time" graph on the screen. The **Return** key functions as both start and stop for each run.
4. Wind the string on the middle step pulley ( $r_{sp} = 2.0$  cm) and using the masses suggested in the data table, click the start button and release the hanging mass. Click stop before the hanging mass hits the floor or before the string completely unwinds and starts rewinding. For each run, measure the hanging mass and record it with its uncertainty.
5. As the disc rotated, the computer recorded the angular velocity of the disc at various times and will show a graph of this data. This graph is a straight line and the slope of this line is the measured angular acceleration. Go to Linear Fit under Analyze on the menu bar. The computer will now exhibit the slope of the line at the bottom of the graph. This is the measured angular acceleration. Record it in the appropriate place in the data table.
6. Repeat steps (4) and (5) three more times obtaining three more data points with the middle step pulley.
7. Repeat steps (4) and (5) using the largest step pulley ( $r_{sp} = 2.5$  cm). Use the masses suggested in the data table and each time record the measured angular acceleration. Now you have collected 8 data points.
8. Turn the disc over so the step pulley is on the bottom and put the steel ring on top. The ring has 2 pins on one side which align with the 2 small holes in the disc. Use the largest step pulley and repeat steps (4) and (5) to collect 4 more data points each time recording the measured angular acceleration.
9. For the very last run of the disc and steel ring combination and step pulley radius  $r_{sp} = 2.5$  cm, record the angular velocity for line number  $n_1 = 5$  and the angular velocity for the largest line number available before the disc stopped accelerating, for example  $n_2 = 155$ .
10. Measure the mass and the radius of the disc and the mass and inner and outer radii of the steel ring (call them  $R_1$  and  $R_2$ ) and record them in the appropriate place in the data table.

## IV. ANALYSIS

1. For each case, calculate the applied torque and record it in the data table. The torque applied to the disc is

$$\tau = Tr_{\text{sp}} \quad (2)$$

where  $T$  is the tension in the string and  $r_{\text{sp}}$  is the radius of the step pulley. Applying Newton's second law to the hanging mass, we find the tension in the string,

$$\begin{aligned} T &= mg - ma \\ &= mg - m\alpha r_{\text{sp}}. \end{aligned} \quad (3)$$

In the above equations, we used the relation  $a = \alpha r_{\text{sp}}$ . Now plugging this expression for  $T$  in the torque equation and doing some algebraic manipulation, we find the following expression for the applied torque

$$\tau = mgr_{\text{sp}} \left( 1 - \frac{\alpha r_{\text{sp}}}{g} \right). \quad (4)$$

For the conditions of this experiment, the quantity  $\alpha r_{\text{sp}}/g \ll 1$  and can be neglected. You do not have to worry about it unless you have measured angular accelerations larger than  $30 \text{ rad/sec}^2$ . Hence we will use the good approximation  $\tau = mgr_{\text{sp}}$  to calculate the torque. In this expression  $m$  is the hanging mass.

2. Calculate the moment of inertia of the disc  $I_{\text{disc}} = \frac{1}{2}M_{\text{disc}}R_{\text{disc}}^2$  and record it in the data table.
3. Calculate the theoretical angular acceleration  $\alpha = \tau/I_{\text{disc}}$  and calculate the percent difference between the experimental and theoretical angular accelerations

$$\% \text{ diff.} = \frac{\alpha_{\text{th}} - \alpha_{\text{exp}}}{\left( \frac{\alpha_{\text{th}} + \alpha_{\text{exp}}}{2} \right)} \times 100. \quad (5)$$

4. Calculate the moment of inertia of the steel ring  $I_{\text{ring}} = \frac{1}{2}M_{\text{ring}}(R_1^2 + R_2^2)$  and enter it in the data table.
5. Calculate the moment of inertia of the combination of the disc and the steel ring  $I_{\text{comb}} = I_{\text{disc}} + I_{\text{ring}}$  and enter it in the data table.
6. For the first 8 data points, plot the torque on the vertical axis and the experimental angular acceleration on the horizontal axis. Draw the best straight line fit for the data and calculate the slope of this straight line. This slope represents the moment of inertia of the disc. Now calculate the percent difference between the slope and the moment of inertia of the disc.

7. For the last 4 data points, plot the torque on the vertical axis and the experimental angular acceleration on the horizontal axis. Draw the best straight line fit for the data and calculate the slope of this straight line and find the percent difference between the slope and the moment of inertia of the combination of the disc and steel ring,  $I_{\text{comb}}$ .
8. Conservation of mechanical energy in rotational motion. From the details of this experiment, it can be shown that "every 10 line numbers recorded by the computer, the string and the hanging mass fall  $\pi$  cm if you are using the 2.5 cm-radius step pulley." Using line numbers  $n_1 = 5$  and for example  $n_2 = 155$ , we find the height through which the hanging mass descends between these two points  $h = h_1 - h_2 = (n_2 - n_1) \left( \frac{\pi}{10} \right) = 0.471$  m. The mechanical energy at line number (5) is

$$\begin{aligned} E_5 &= K_5 + U_5 \\ &= \frac{1}{2} I_{\text{comb}} \omega_5^2 + \frac{1}{2} m v_5^2 + mgh. \end{aligned} \quad (6)$$

and the mechanical energy at line number 155 is

$$\begin{aligned} E_{155} &= K_{155} + U_{155} \\ &= \frac{1}{2} I_{\text{comb}} \omega_{155}^2 + \frac{1}{2} m v_{155}^2. \end{aligned} \quad (7)$$

In the above equations the linear velocity  $v$  and the angular velocity  $\omega$  are related by  $v = r_{\text{sp}} \omega$  where  $r_{\text{sp}}$  is the step pulley radius = 0.025 m. Calculate  $E_5$  and  $E_{155}$ .

9. Calculate the percent difference between  $E_5$  and  $E_{155}$

$$\% \text{ diff.} = \frac{E_5 - E_{155}}{\left( \frac{E_5 + E_{155}}{2} \right)} \times 100. \quad (8)$$

10. Write a conclusion summarizing your results. Comment on the success of this experiment. Are your results consistent with the theory? Explain any percent differences which are larger than 10%. What do you think are the two most important sources of error in this experiment?

<b>Experiment (7) Data Table (1)</b>
$M_{\text{disk}} \pm \delta M_{\text{disk}} =$
$R_{\text{disk}} \pm \delta R_{\text{disk}} =$
$I_{\text{disc}} = (1/2)M_{\text{disc}}R_{\text{disc}}^2 =$
$M_{\text{ring}} \pm \delta M_{\text{ring}} =$
$R_1 \pm \delta R_1 =$
$R_2 \pm \delta R_2 =$
$I_{\text{ring}} = (1/2)M_{\text{ring}}(R_1^2 + R_2^2) =$
$I_{\text{comb}} = I_{\text{disc}} + I_{\text{ring}} =$
$\omega_5 =$
$\omega_{155} =$
$E_5 =$
$E_{155} =$

Experiment (7) Data Table (2)					
Run	Hanging Mass $M$ (kg)	Applied Torque $\tau = mgr_{sp}$ (Nm)	Experimental Angular Acceleration $\alpha_{exp}$ (rad/s <sup>2</sup> )	Theoretical Angular Acceleration $\alpha_{th} = \tau/I$ (rad/s <sup>2</sup> )	% Difference
$r_{sp} = 0.020$ m					
1	0.055				
2	0.105				
3	0.155				
4	0.205				
$r_{sp} = 0.025$ m					
5	0.055				
6	0.105				
7	0.155				
8	0.205				
Disc + Steel Ring, $r_{sp} = 0.025$ m					
9	0.055				
10	0.105				
11	0.155				
12	0.205				