Beginnings of Trigonometry

Trigonometry is an area of mathematics used for determining geometric quantities. Its name, first published in 1595 by B. Pitiscus, means "the study of trigons (triangles)" in Latin. Ancient Greek Mathematicians first used trigonometric functions with the chords of a circle. The first to publish these chords in 140 BC was Hipparchus, who is now called the founder of trigonometry. In AD 100, Menelaus, another Greek mathematician, published six lost books of tables of chords. Ptolemy, a Babylonian, also wrote a book of chords. Using chords, Ptolemy knew that

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

and

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

In his great astronomical handbook, *The Almagest,* Ptolemy provided a table of chords for steps of $\frac{1}{2}°$, from 0° to 180°, that is accurate to 1/3600 of a unit. Ptolemy's degree of accuracy is due in part because the Hellenistic Greeks had adopted the Babylonian base-60 (sexagesimal) numeration system. He also explained his method for constructing his table of chords, and in the course of the book he gave many examples of how to use the table to find unknown parts of triangles from known parts. Ptolemy provided what is now known as Menelaus's theorem for solving spherical triangles, as well, and for several centuries his trigonometry was the primary introduction to the subject for any astronomer.

Sine first appeared in the work of Aryabhata, a Hindu. He used the word jya for sine. He also published the first sine tables. Brahmagupta, in 628, also published a table of sines for any angle. Jya became jiba in translation and jiba became jaib in later writings. Jaib means fold in Arabic. This was translated into sinus, or fold in Latin. In 1533, Regiomontanus' published De triangulis omnimodis which dealt with planar trigonometry and inverses. Later, Rheticus published Copernicus' book dealing with Trigonometry in Astronomy in 1542. Edmund Gunter first used the abbreviation sin in 1624. Sin was first used in a book in 1634. Other variances for cosine and tangent were also still very popular, especially among different languages. Although sine, cosine, and tangent were used very much by astronomers and surveyors, the functions secant and cosecant were of little use to these practical minded mathematicians.

Late in the 8th century, Muslim astronomers inherited both the Greek and the Indian traditions, but they seem to have preferred the sine function. By the end of the 10th century they had completed the sine and the five other functions and had discovered and proved several basic theorems of trigonometry for both plane and spherical triangles. Several mathematicians suggested using $r = 1$ instead of $r = 60$; this exactly produces the modern values of the trigonometric functions. The Muslims also introduced the polar

triangle for spherical triangles. All of these discoveries were applied both for astronomical purposes and as an aid in astronomical time-keeping and in finding the direction of Mecca for the five daily prayers required by Muslim law. Muslim scientists also produced tables of great precision. For example, their tables of the sine and tangent, constructed for steps of 1/60 of a degree, were accurate for better than one part in 700 million. Finally, the great astronomer Nasir al-Din al-Tusi wrote the *Book of the Transversal Figure*, which was the first treatment of plane and spherical trigonometry as independent mathematical sciences.

The Latin West became acquainted with Muslim trigonometry through translations of Arabic astronomy handbooks, beginning in the 12th century. The first major Western work on the subject was written by the German astronomer and mathematician Johann Müller, known as Regiomontanus. In the next century the German astronomer Georges Joachim, known as Rheticus introduced the modern conception of trigonometric functions as ratios instead of as the lengths of certain lines. The French mathematician François Viète introduced the polar triangle into spherical trigonometry, and stated the multiple-angle formulas for sin(nq) and cos(nq) in terms of the powers of sin(q) and cos(q).

Trigonometric calculations were greatly aided by the Scottish mathematician John Napier, who invented logarithms early in the 17th century. He also invented some memory aids for ten laws for solving spherical triangles, and some proportions (called Napier's analogies) for solving oblique spherical triangles.

Almost exactly one half century after Napier's publication of his logarithms, Isaac Newton invented the differential and integral calculus. One of the foundations of this work was Newton's representation of many functions as infinite series in the powers of x. Thus Newton found the series sin(x) and similar series for $\cos x$ and $\tan x$. With the invention of calculus, the trigonometric functions were taken over into analysis, where they still play important roles in both pure and applied mathematics.

Finally, in the 18th century the Swiss mathematician Leonhard Euler defined the trigonometric functions in terms of complex numbers This made the whole subject of trigonometry just one of the many applications of complex numbers, and showed that the basic laws of trigonometry were simply consequences of the arithmetic of these numbers.

Trigonometry has been used throughout modern and ancient history dealing with practical applications, such as surveying. Modernly, it has incorporated many other ideas instead of just triangles. Without their discovery, surveyors and other practical mathematicians would not be able to efficiently determine relationships.