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# Tutor with Vision Training Part 3: Part 3: Bridging the Void

Abel Villarreal, M. Ed in mathematics  
Teaching & Learning Center (TLC)  
Austin Community College



# A Quick Review of Part 1

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In Part 1, you learned how the brain learns and is conditioned to learn. You also explored the types of learners (auditory, tactile, and visual) and their innate types of intelligences that create a learning “web.” Lastly, you were asked to ponder on YOUR learning style and YOUR types of intelligences and how they affect student learning.



## A Quick Review of Part 2

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In Part 2, you learned how data is used to diagnose student academic weaknesses, set up a student “success plan,” and monitor student progress.



## A Quick Overview of Part 3

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In Part 3, you will learn how to write curriculum that helps students connect with what is being taught quickly, efficiently, and with little frustration.



Taking a student from point **A** (say what?) to point **B** (“GOT IT!”) is the essence of being an effective tutor. A tutor like a teacher must push the limits of learning while maintaining a solid mathematical base on which to build future knowledge.



# The Great Void

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Somewhere between course expectations and student psyches there is a mental space I call the “Great Void.” Depending on a student’s mathematical foundation this space can be vast or small. To bridge this “void,” the teacher must find an anchor point in the student’s mind and another anchor point in the course curriculum and build a bridge between both points. This undertaking is easier said than done.



# Crossing the Void

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To bridge this space, one begins by finding how deep the students' understanding of mathematical ideas is. Key math words and ideas are then replaced with more familiar and friendlier equivalent ones. Ditto for the computational process. A tutor reinforces concepts and skills and then scaffolds all the more formal terms until the "void" is no more.

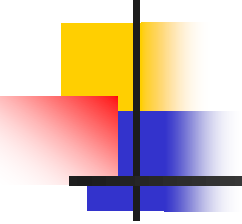
# TAKS Objective 1: Numbers, Operations, and Quantitative Reasoning



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(9th Grade, April 2004)

#49. Ms. Hill wants to carpet her rectangular living room, which measures 14 feet by 11 feet. If the carpet she wants to purchase costs \$1.50 per square foot, including tax, how much will it cost to carpet her living room?



# TAKS Objective 1: Numbers, Operations, and Quantitative Reasoning (rewritten)

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#49. Ms. Hill has a rectangular living room that measures 14 feet by 11 feet. A square foot of carpet costs \$1.50 to install, including tax. How much will it cost to carpet the living room?

Solution:  $\text{Area} = L \cdot W$   
 $= 14 \cdot 11$   
 $= 154 \text{ square feet}$

Total cost = (total square footage) • (\$1.50 per square foot)  
 $= (154) \cdot (1.50)$   
 $= \$231$



# TAKS Objective 1: Numbers, Operations, and Quantitative Reasoning

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(9th Grade, April 2006)

#25. In many parade floats, flowers are used to decorate the floats. The table at right shows the number of flowers used in each row of a parade float. Which equation best represents the data?

Row #, r	# of flowers, n
1	54
2	58
3	62
4	66

- A  $n = 2r + 52$
- B  $n = r + 54$
- C  $n = 4r + 50$
- D  $n = 4r + 4$



# TAKS Objective 1: Numbers, Operations, and Quantitative Reasoning (Rewritten)

#78. Flowers are often used to decorate parade floats. The table at right shows the number of flowers used in each row of a parade float. Which equation best represents the data?

Row #	# of flowers
1	54
2	58
3	62
4	66
5	70
$r$	

- A:  $2r + 52$
- B:  $r + 54$
- C:  $4r + 50$
- D:  $4r + 4$

**Solution:** Notice that the number of flowers increase by 4 at every row from the previous row. One way to figure the correct formula is to “plug in” 1, then 2, then 3, and so on to see which formula will generate all the correct answers (54, 58, etc.) Another way is to plug in data into a graphing calculator as List1 (L1) and List 2 (L2) and do a linear regression (LinReg). The correct answer is “C.”

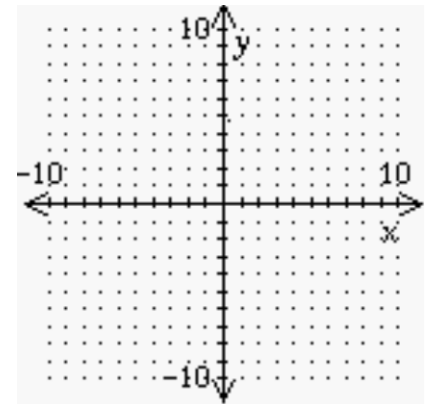
# TAKS Objective 6: Geometry & Spatial Reasoning

(9th Grade, April 2004)

#18.  $\triangle DFG$  has vertices at  $D(2, 4)$ ,  $F(4, 8)$ ,  $G(6, 4)$ .

$\triangle DFG$  is dilated by a scale of  $1/4$  and has the origin  $(0, 0)$  as the center of dilation. What are the coordinates of  $F'$ ?

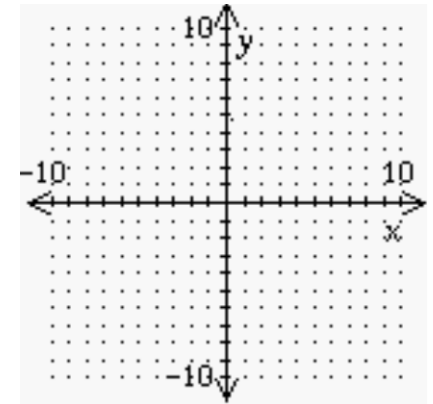
- A (1, 2)
- B  $(1/2, 1)$
- C (16, 32)
- D  $(3/2, 1)$



# TAKS Objective 6: Geometry & Spatial Reasoning (rewritten)

#18. Triangle DFG has vertices (corners) at  $D(2, 4)$ ,  $F(4, 8)$ ,  $G(6, 4)$ . Triangle DFG is dilated (made larger or smaller) by a scale of one-fourth and has the origin  $(0, 0)$  as the center of dilation. What are the coordinates of  $F'$  ( $F$  prime)?

- A.  $(1, 2)$
- B.  $(0.5, 1)$
- C.  $(16, 32)$
- D.  $(1.5, 1)$



**Solution:** Graphing the points  $D$ ,  $F$ , and  $G$  will give you a visual image of what you are given. You now have to imagine the same image one-fourth the size. The easiest thing to do is zero-in on coordinate  $F$  and multiply each coordinate by one-fourth and get  $(1, 2)$ . The answer is “A.”

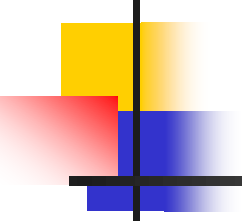


## TAKS Objective 8: Measurement and Similarity

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(9th Grade, April 2004)

#23. A cylindrical water tank has a radius of 2.8 feet and a height of 5.6 feet. The tank is filled to the top. If water can be pumped out at a rate of 36 cubic feet per minute, about how long will it take to empty the water tank?



## TAKS Objective 4: Measurement and Similarity (rewritten)

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#23. A cylindrical water tank has a radius of 2.8 feet and a height of 5.6 feet. The tank is filled to the top. If water can be pumped out at a constant rate of 36 cubic feet per minute, about how long (minutes) will it take to empty the tank? ( $\pi \approx 3.14$ )

Solution: First compute the cylinder's volume using  $\pi r^2 h$  (see TAKS formula chart). Then you have to imagine draining out groups of 36 until the tank is empty.

$$\begin{aligned}\text{Volume} &= (3.14) \cdot (2.8)^2 (5.6) \\ &\approx 137.9 \text{ cubic feet}\end{aligned}$$

$$\begin{aligned}\text{Time to drain} &= \text{volume} \div \text{rate of drainage} \\ &= 137.9 \div 36 \\ &\approx 3.8 \text{ minutes or about 4 minutes}\end{aligned}$$



# TAKS Objective 9: Percents, and Probability Statistics

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(9th Grade, April 2006)

- #45. A jar contains 6 red marbles and 10 blue marbles, all of equal size. If Dominic were to randomly select one marble without replacement and then select another marble from the jar, what would be the probability of selecting 2 red marbles from the jar?



# TAKS Objective 9: Percents, and Probability Statistics (rewritten)

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#45. A jar contains 6 red marbles and 10 blue marbles, all of equal size. If Darren randomly selects one marble without replacement and then selects a second marble from the jar, what is the probability of selecting 2 red marbles from the jar?

Solution: The word “and” between two marble selections imply multiplication. Without replacement means that the marble does NOT go back in the jar. Selecting a red marble on the first pick is 6 red marbles out of 16 (total) marbles. Since the marble is not put back in the jar, you have 15 marbles in the jar. Selecting a second red marble is 5 red marbles out of 15 (new total) marbles.

Probability of selecting two red marbles  $= (6/16) \cdot (5/15) = 1/8$ .

[Don't forget to reduce fractions.]



# TAKS Objective 10: Mathematical Processes & Tools

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(9th Grade, April 2004)

#15. Mr. Collins invested some money that will double in value every 12 years. If he invested \$5,000 on the day of his daughter's birth, how much will the investment be worth on his daughter's 60th birthday?

- (A) \$300,000
- (B) \$160,000
- (C) \$80,000
- (D) \$320,000

# TAKS Objective 10: Mathematical Processes & Tools (rewritten)

#15. Mr. Campos invested some money that will double in value every 12 years. If he invested \$5,000 on the day of his son's birth, how much will the investment be worth on the son's 60th birthday?

Solution: You can apply all sorts of algebraic tricks to this problem, but the easiest, most visual method is best. Consider:

Birth	\$5000
12 years later	\$10,000(money doubles)
$12 + 12 = 24$ years	\$20,000(money doubles)
$12 + 12 + 12 = 36$ years	\$40,000(money doubles)
$12 + 12 + 12 + 12 = 48$ years	\$80,000(money doubles)
$12 + 12 + 12 + 12 + 12 = 60$ years	\$160,000

(Notice there are NO multiple choices!)



# Assessment 1

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- Download a sample STAAR, TAKS released test or study materials from [www.tea.state.tx.us/student.assessment/released-tests/](http://www.tea.state.tx.us/student.assessment/released-tests/)
- Select 5 test items of your choice on any subject and any grade.
- Rewrite the test items like the examples shown.
- Submit them as a pdf or Word file.



# Elements of Rewriting Curriculum

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- The rewritten product needs to connect to ALL students (Tier 1, 2, 3).
- Key word replacements must be in the students' vocabulary.
- Create bridges from key words to equivalents.



# Tier 1 Learners

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- Tier 1 students are usually the top performing ones. They are quick learners, get the assignment done, and ready to move to the next task with little to no teacher help (nerds, honor roll students).
- Tier 1 students usually function/learn at two or more learning style levels simultaneously (visual, auditory, kinesthetic).



# Tier 2 Learners

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- Tier 2 students will struggle a bit, but with a little instruction and practice they will successfully complete assigned tasks (average students).
- Tier 2 students will usually function and learn at one of the three learning style levels.



# Tier 3 Learners

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- Tier 3 students have no clue as to what the task is, how to do it, or when it's due. Often they do not care whether or not they get a zero and tend to divert attention away from their academic deficiencies by acting up or being disciplinary problems.
- Tier 3 students have little hope or desire to improve their status. Their academic skills are in of need “intensive care” intervention (dropouts, chronic absentees, zeros).



# Good news for Tier 3 Learners

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Tier 3 students are usually kinesthetic learners at the beginning of their remediation and many add another learning style level later. To “reach” Tier 3s, one needs to use lots of visual or kinesthetic clues and models.



# Rewriting a Lesson

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- Look over the lesson and pinpoint areas that students missed.
- Determine whether or not the students understood the directions and vocabulary.
- Determine whether the students made careless errors or had no clue what to do.
- Determine students' skills level on the vocabulary and language of sections missed and rewrite instructions and problems accordingly. Examples in key spots may be necessary.
- Rewrite **ONLY** parts that are necessary
- Build new problems/tasks from easy to challenging.
- Don't overdo the rewrite. **LESS is MORE!**



# Connect to TEKS

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- Determine which TEKS objectives are connected to the rewritten lesson and weave a few STAAR or TAKS problems into the lesson.
- Start with a few problems and increase the difficulty and number until you reach a balance. Use released test problems.



# Re-evaluate Rewritten Lesson

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- Once the rewritten lesson is mastered, prepare a short quiz to confirm mastery of concepts on rewritten lesson. If successful, students are ready for a new task.
- If not successful, rewrite again using the same criteria.

# What the Textbook Says

1-2

## Adding and Subtracting Real Numbers

**Objectives**  
Add real numbers.  
Subtract real numbers.

**Vocabulary**  
absolute value  
opposites  
additive inverse

### Why learn this?

The total length of a penguin's dive can be determined by adding real numbers. (See Example 4.)

All the numbers on a number line are called *real numbers*. You can use a number line to model addition and subtraction of real numbers.

**Addition**  
To model addition of a positive number, move right. To model addition of a negative number, move left.

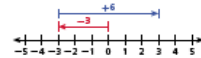
**Subtraction**  
To model subtraction of a positive number, move left. To model subtraction of a negative number, move right.



### EXAMPLE 1 Adding and Subtracting Numbers on a Number Line

Add or subtract using a number line.

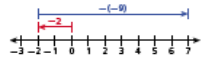
**A**  $-3 + 6$



Start at 0. Move left to  $-3$ .  
To add 6, move right 6 units.

$-3 + 6 = 3$

**B**  $-2 - (-9)$

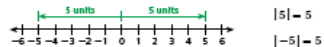


Start at 0. Move left to  $-2$ .  
To subtract  $-9$ , move right 9 units.

$-2 - (-9) = 7$

**CHECK IT OUT!** Add or subtract using a number line.  
1a.  $-3 + 7 = 4$     1b.  $-3 - 7 = -10$     1c.  $-5 - (-6.5) = 1.5$

The **absolute value** of a number is its distance from zero on a number line. The absolute value of 5 is written as  $|5|$ .



**Know it!**  
Note

### Adding Real Numbers

WORDS	NUMBERS
<b>Adding Numbers with the Same Sign</b> If two numbers have the same sign, add their absolute values and use the sign of the numbers.	$3 + 6 = 9$ $-2 + (-9) = -11$ $3 + 6 = 9$ $2 + 9 = 11$
<b>Adding Numbers with Different Signs</b> If two numbers have different signs, find the difference of their absolute values and use the sign of the number with the greater absolute value.	$-8 + 12 = 4$ $3 + (-15) = -12$ $12 - 8 = 4$ $15 - 3 = 12$ $ 12  >  -8 $ $ -15  >  3 $

### EXAMPLE 2 Adding Real Numbers

Add.

**A**  $-3 + (-16)$

$-3 + (-16)$

$-19$

When the signs of the numbers are the same, find the sum of their absolute values:  $3 + 16 = 19$ . Both numbers are negative, so the sum is negative.

**B**  $x + 7$  for  $x = -13$

$x + 7 = (-13) + 7$

$(-13) + 7$

$-6$

First substitute  $-13$  for  $x$ .  
When the signs are different, find the difference of the absolute values:  $13 - 7 = 6$ .  
Use the sign of the number with the greater absolute value. The sum is negative.

**CHECK IT OUT!**

Add.

2a.  $-5 + (-7)$

$-12$

2b.  $-13.5 + (-22.3)$

$-35.8$

2c.  $x + (-68)$  for  $x = 52$

$-16$

Two numbers are **opposites** if their sum is 0. A number and its opposite are the same distance from zero. They have the same absolute value.

A number and its opposite are **additive inverses**. To subtract signed numbers, you can use additive inverses. Subtracting a number is the same as adding the opposite of the number.

Additive Inverses

$$11 - 6 = 5 \quad 11 + (-6) = 5$$

Subtracting 6 is the same as adding the inverse of 6.

**Know it!**  
Note

### Subtracting Real Numbers

WORDS	NUMBERS	ALGEBRA
To subtract a number, add its opposite. Then follow the rules for adding signed numbers.	$3 - 8 = 3 + (-8)$ $8 - 3 = 5$ $ -8  >  3 $ $-5$	$a - b = a + (-b)$

# What the Textbook Says

## EXAMPLE 3 Subtracting Real Numbers

Subtract.

**A**  $7 - 10$   
 $7 - 10 = 7 + (-10)$

To subtract 10, add  $-10$ .  
 When the signs of the numbers are different, subtract the absolute values:  $10 - 7 = 3$ . Use the sign of the number with the greater absolute value. The sum is negative.

$-3$

**B**  $-3 - (-12)$   
 $-3 - (-12) = -3 + 12$

To subtract  $-12$ , add 12.  
 When the signs of the numbers are different, subtract the absolute values:  $12 - 3 = 9$ . Use the sign of the number with the greater absolute value. The sum is positive.

$9$

**C**  $x - 22$  for  $x = -11$   
 $x - 22 = -11 - 22$   
 $-11 + (-22)$

First substitute  $-11$  for  $x$ .  
 To subtract 22, add  $-22$ .  
 When the signs of the numbers are the same, add the absolute values:  $11 + 22 = 33$ . Both numbers are negative, so the sum is negative.

$-33$

### Helpful Hint

On many scientific and graphing calculators, there is one button to express the opposite of a number and a different button to express subtraction.



Subtract.  
 3a.  $13 - 21 = -8$    3b.  $\frac{1}{2} - (-3\frac{1}{2}) = 4$    3c.  $x - (-12)$  for  $x = -14$   
 $-2$

## EXAMPLE 4 Biology Application

An emperor penguin stands on an iceberg that extends 10 feet above the water. Then the penguin dives to an elevation of  $-67$  feet to catch a fish.

What is the total length of the penguin's dive?



$10 - (-67)$

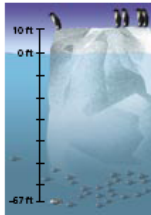
$10 - (-67) = 10 + 67$   
 $= 77$

To subtract  $-67$ , add 67.  
 Find the sum of the absolute values.

The total length of the penguin's dive is 77 feet.



4. **What If...?** The tallest known iceberg in the North Atlantic rose 550 feet above the ocean's surface. How many feet would it be from the top of the tallest iceberg to the wreckage of the *Titanic*, which is at an elevation of  $-12,468$  feet? **13,018 ft**



### THINK AND DISCUSS

1. The difference of  $-7$  and  $-3$  is  $-2$ . Explain why the difference is greater than  $-7$ .

2. **CONJECTURE** Copy and complete the graphic organizer. For each pair of points, add absolute the sum and the difference of the first point and the second point as positive or negative.

Point 1	Point 2	Difference
7	3	
3	7	
-7	-3	
-3	-7	

## 1-2 Exercises

### GUIDED PRACTICE

1. **Wordology** The sum of a number and its \_\_\_\_\_ is always zero. Opposite or absolute value? **opposite**

101 **EXAMPLE 1** Add or subtract real numbers. See. p. 14.   1.  $-4 + 7 = 3$    2.  $-12 - 3 = -15$    3.  $3.8 - 2.2 = 1.6$    4.  $3.8 - 2.2 = 1.6$    5.  $3 - (-4) = 7$

102 **EXAMPLE 2** Add.   1.  $4.71 + 1.10 = 5.81$    2.  $4\frac{1}{2} + (-2\frac{1}{2}) = 2$    3.  $10.8 + a$  for  $a = -17.2 = -6.4$

103 **EXAMPLE 3** Subtract.   1.  $32 - 28 = 4$    2.  $6.2 - 8.4 = -2.2$    3.  $11 - a$  for  $a = 3 = 8$    4.  $1\frac{1}{2} - 2\frac{1}{2} = -1$

104 **EXAMPLE 4** Subtract. The Clayco Iceberg Storage (CIS) reports the average volume of icebergs for 30 consecutive years. Use the table to determine the total decrease in the CIS for the two days. **-558.38**

Year	Volume (cubic meters)
2007	18.5
2008	1.2
2009	2.1
2010	3.2
2011	4.3
2012	5.4
2013	6.5
2014	7.6
2015	8.7
2016	9.8
2017	10.9
2018	12.0
2019	13.1
2020	14.2
2021	15.3
2022	16.4
2023	17.5
2024	18.6
2025	19.7
2026	20.8
2027	21.9
2028	23.0
2029	24.1
2030	25.2

### PRACTICE AND PROBLEM SOLVING

Add or subtract real numbers. See. p. 14.

15.  $-3 + 9 = 6$    16.  $6 + (-12) = -6$    17.  $\frac{1}{2} - 12 = -11\frac{1}{2}$    18.  $-\frac{2}{3} + 8 = 7\frac{1}{3}$

19.  $12 - 18 = -6$    20.  $12 - 18 = -6$    21.  $-18 + (-12) = -30$    22.  $16 - 2.2 = 13.8$    23.  $16 - 2.2 = 13.8$

24.  $12 - 22 = -10$    25.  $12 - 22 = -10$    26.  $12 - a$  for  $a = 24.8 = -12.8$    27.  $12 - a$  for  $a = 24.8 = -12.8$

28.  $12 - 22 = -10$    29.  $12 - 22 = -10$    30.  $12 - a$  for  $a = 24.8 = -12.8$    31.  $12 - a$  for  $a = 24.8 = -12.8$

32.  $12 - 22 = -10$    33.  $12 - 22 = -10$    34.  $12 - a$  for  $a = 24.8 = -12.8$    35.  $12 - a$  for  $a = 24.8 = -12.8$

### Answers to Think and Discuss

Possible answers:  
 1. Subtracting a negative number is the same as adding a positive number. This means you move right on the number line, making the sum greater than the first addend.  
 2. See p. 13.

## 1-2 Exercises

### Assignment Guide

Assign Guided Practice exercises as necessary.

If you finished **Example 1**:  
 Basic: 15–18, 24–27  
 Average: 19–21, 24, 26, 35, 44  
 Advanced: 33–35, 39–44

If you finished **Example 2**:  
 Basic: 15–18, 34–42, 46, 49, 51, 54, 59–62  
 Average: 19–21, 24–26, 40, 43, 46, 49–51, 54, 59–62  
 Advanced: 33–35, 29–32, 42, 45, 52, 55, 63–66, 69–74

**Homework Check**  
 Quality check key concepts.  
 Exercises: 16, 18, 22, 24, 43, 45

### TAKE PRACTICE

Exercise	Location
101	16, 17–18, 19, 23, 28
102	18, 20–21, 24, 25, 30–32
103	19–21, 24–26, 30–32, 35, 44
104	20–22, 24–26, 30–32, 35, 44
105	20–22, 24–26, 30–32, 35, 44
106	18, 19–21, 24, 25, 30–32

Lesson 1-2 17

# What the Textbook Says

**EXAMPLE** Exercise on numbers applying the subtraction of numbers to debt. This exercise prepares students for the Multi-Step 1&2 Prep on page 8.

**Answers**  
 a. 1000 B  
 b. 1000 B  
 c. 1000 B; because subtracting a negative number is the same as adding a positive number.

**1.0 PRACTICE A**  
**1.0 PRACTICE C**  
**1.0 PRACTICE B**

**2.0** Chapter 7 Foundations for Algebra

**1.0** Evaluate the expression  $a + (-3)$  for each value of  $a$ .

24.  $a = 312$  **267** 25.  $a = 3.75$  **0.75** 26.  $a = -\frac{1}{2}$   **$-\frac{1}{2} - 3 = -3\frac{1}{2}$**  27.  $a = -\frac{1}{2}$   **$-\frac{1}{2} - 3 = -3\frac{1}{2}$**

28.  $-8 - 3 = -11$  29.  $-2 + (-12) = -14$  30.  $18 - (-5) = 23$  31.  $120 - 42 = 78$   
 32.  $3.2 - 2.3 = 0.9$  33.  $-4.7 - (-6.5) = 1.8$  34.  $\frac{2}{3} - \frac{1}{8} = \frac{13}{24}$  35.  $\frac{2}{3} - \frac{1}{8} = \frac{13}{24}$

**2.0** **Business** A restaurant manager lost \$12 in business during the month of January. Business picked up in February, and he ended that month with a profit of \$200.

**3.** What was the manager's profit after January and February? **\$188**

**4.** What was the restaurant's profit after January, February, and March? **\$200**

**Compare.** Write  $>$ ,  $=$ , or  $<$ .

37.  $-4 - (-4)$   **$0 > -2$**  38.  $(-10)(2)$   **$0 < -20$**  39.  $3 - (-2)$   **$5 > -2$**   
 40.  $-2 - 8$   **$-10 < 11$**  41.  $1 - 10 + 3$   **$-6 < -12$**  42.  $2 + (-4)$   **$-2 < -12$**

**43. 2.0** **Travel** Death Valley National Park is located in California. Use the table to determine the difference in elevation between the highest and lowest locations.

Death Valley National Park	
Location	Elevation (ft)
Tataviam	-28
Badwater	-282
California's Great Basin	-100
California's Great Dunes	112

**Critical Thinking** Use examples to explain whether each statement is sometimes, always, or never true.

44. The difference between two negative numbers is positive. **sometimes**  
 45. The sum of two negative numbers is negative. **always**  
 46. The difference of a negative number and a positive number is negative. **never**

**47. JUNCTION ANALYSIS** Which is less? Explain the sum.

**A: the opposite of -4 does not have been added.**

**48.** This problem will prepare you for the Multi-Step 1&2 Prep on page 8.

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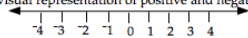
# What A Teacher Might Say

Name \_\_\_\_\_  
A. Villarreal, teacher, 1 of 2

## Algebra Ia Lesson 1-2 The Number Line

Date \_\_\_\_\_  
Text pages 14 -16

A number line is a visual representation of positive and negative numbers.

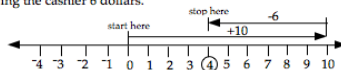


Historically, \_\_\_\_\_ numbers are to the left of zero and \_\_\_\_\_ numbers are to the right of zero. Zero is considered neutral. For convenience, the "+" sign on the positive numbers on the number line are dropped. The farther to the left you go, the smaller the number. The farther to the right you go, the larger the number.

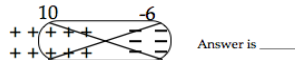
Key words in the English language add a positive or negative sign to a number. For example, a drop of 10° means -10. Listed below are the more common key words.

Negative	Positive	Examples
loss	gain or profit	a gain of \$7 = +7 or 7
drop	rise	a 3° drop = -3
dive	climbs	a plane climbs 50 ft = _____
increase	decrease	taxes increased 12% = _____
less	more	less \$1.20 = _____
plus	minus	plus \$3.40 tax = 3.40
subtract	add	add a \$3 charge = 3
below sea level	above sea level	sub dives 100 ft = -100
discount		discount of \$5 = _____

Signed numbers lead us to think about addition and subtraction. For example, Joe has \$10 and spends \$6. We can imagine Joe opening his wallet having 10 dollars, pulling out six \$1 bills to pay for his purchase and have \$4 left. On a number line, we can "see" Joe moving 10 spots to the \_\_\_\_\_ and stop. Now Joe goes to the \_\_\_\_\_ 6 spots to represent paying the cashier 6 dollars.



Another way to represent  $10 + (-6)$  is to think of ten little "+" signs for 10 and six little "-" signs for (-6). Cross out/cancel one of each sign until you run out of either "+" or "-" signs and 4 "+" signs. In other words,



Name \_\_\_\_\_  
A. Villarreal, teacher, 2 of 2

Date \_\_\_\_\_  
Text pages 14 -16

## Algebra Ia Lesson 1-2 The Number Line

There are two rules for adding and subtracting numbers.

- RULE 1:** When numbers are the same sign, keep the sign and **ADD** quantities.
- RULE 2:** When integers differ in sign, **SUBTRACT** quantities and keep sign of the larger quantity.

Can you see the connection between a number line and the operations of addition and subtraction? The use of the addition operation (+) and subtraction operation (-) is just a shortcut for number lines. Using operations also allows one to easily add or subtract large numbers as well as fractions and decimals without the mess of drawing all kinds of lines.

Examples:  $3 + 6 = 9$  [Rule 1]       $^{-}6 + ^{-}2 = ^{-}8$  [Rule \_\_\_\_]  
 $-6 + 2 = (2 - 6) = \underline{\hspace{1cm}}$  [Rule 2]       $-4 + 11 = (11 - 4) = 7$  or +7 [Rule \_\_\_\_]

**Try these problems:**  
1. Classify "the price of gasoline increased \$0.70" into a signed number.

2. Draw a number line for  $-7 + 4$ .

3. Use the "signs" (+ and -) to calculate  $^{-}6 + 9$

4. Use addition and subtraction to calculate  $^{-}\$34.50 + \$81.75$

# What A Teacher Might Say

Name \_\_\_\_\_

## The Number Line Algebra 1

<p><b>Classify as a positive or negative number.</b></p> <p>1. a balloon climbs 140 ft.</p> <p>2. A share of stock lost \$3 in value.</p> <p>3. The temperature dropped 10°C.</p> <p>4. A submarine dives 240 ft.</p>	<p><b>Compute by ANY method.</b></p> <p>15. <math>(-68) + 37</math></p> <p>16. <math>100 + (-74) + (-16)</math></p> <p>17. <math>(-34) + (-12) + 45 + (-24)</math></p>	<p>21. The TexPro company posted the following stock information.</p> <p>Fri close at \$21.40 a share. Mon gained \$0.43 a share Tues lost \$0.26 a share Wed gained \$0.35 a share Thurs gained \$0.49 a share Fri lost \$0.17 a share.</p>
<p><b>Draw a number line and compute result.</b></p> <p>5. <math>5 + (-3)</math></p> <p>6. <math>-3 + (-6)</math></p> <p>7. <math>-2 + 8</math></p> <p>8. <math>0 + (-4)</math></p>	<p><b>Solve the following.</b></p> <p>18. What is the correct change when you buy three \$12 shirts with a \$50 bill?</p> <p>19. A submarine cruises at 150 below sea level. It then dives 75 feet and stays at this depth for an hour. It then rises 24 feet and holds steady. What is the submarine's final cruise depth?</p>	<p>(a) State the value of one share of TexPro stock at the end of each day.</p> <p>Mon. Tues. Wed. Thurs. Fri.</p> <p>(b) A share of TexPro stock was at its highest value on which day?</p>
<p><b>Compute the following using "+" and "-" signs.</b></p> <p>9. <math>-5 + 8</math></p> <p>10. <math>(-4) + (-3)</math></p> <p>11. <math>12 - 8</math></p> <p>12. <math>3 + (-12)</math></p>	<p>20. Andy's bank account has \$145. He writes checks for \$74 and \$58. He then deposits \$347. What is Andy's final bank balance?</p>	<p>22. Write a problem for these signed numbers: -5, 12, -3, and 4.</p>
<p><b>Compute by ANY method.</b></p> <p>13. <math>36 + (-27)</math></p> <p>14. <math>(-48) + (-59)</math></p>		



# Where Do I Begin to Learn How to Rewrite Lessons?

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This skill takes lots of practice. Before you set off in your own “lesson rewriting” quest:

- Emulate lesson writing from teachers you trust.
- Edit website materials that are teacher/student friendly and copyright free, and make them your own ([www.purplemath.com](http://www.purplemath.com)).
- Borrow/use materials from teachers you trust.



# Starting Points

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- Explore the websites below for possible sources of lessons you can use.

[http://www.internet4classrooms.com/online\\_powerpoint.htm](http://www.internet4classrooms.com/online_powerpoint.htm)

<http://www.worldofteaching.com>

[www.purplemath.com](http://www.purplemath.com)



# How Far do I Need to go?

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Rewriting a lesson (or part of it) may be enough to bridge the void, but sometimes a teacher must also rewrite a part or all the homework lesson. Since such an undertaking may take lots of time and effort, it would be best if the teacher teams up with other teachers and share the work and the final product.



# Closing Thought

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Most challenging students like a good story or movie. Long term learning follows the story line. Story details and embellishments are added to the story as the student becomes more and more connected to the story. Mathematics is learned by **DOING!** You would not read the dictionary before you read a good book or see a movie based on the book. Mathematics is no different.



# Assessment 2

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- Review “8.6text.pdf” and “8.6tchrlesn.pdf” side by side. Which lesson version would you like to use with your students? Why?
- Select a lesson of your choice from a textbook, lecture, or presentation on any subject. Rewrite it so that ALL students can easily connect with it.
- Write an alternate homework assignment for the lesson.
- Submit your final products in pdf or Word format.