

## Become Friends with Logarithms

Nothing like lots of practice to become proficient at this stuff! Please fill in this sheet, and practice, practice, practice!!!

**Rewriting exponential expressions into equivalent logarithmic expressions.**

General rule:  $y = b^x$  is the same as  $x = \log_b y$

### **I. Rewrite these exponentials into logarithmic form.**

Example:  $3^4 = 81$  becomes  $\log_3 81 = 4$

- |                    |                     |
|--------------------|---------------------|
| 1) $4^3 = 64$      | 2) $2^7 = 128$      |
| 3) $3^1 = 3$       | 4) $5^0 = 1$        |
| 5) $8^2 = 64$      | 6) $3^{-1} = 1/3$   |
| 7) $4^{-2} = 1/16$ | 8) $10^5 = 100,000$ |
| 9) $3^3 = 27$      | 10) $e^5 = 148.413$ |

### **II. Rewrite these logarithms into exponential form. See above example.**

- |                      |                       |
|----------------------|-----------------------|
| 1) $\log_2 8 = 3$    | 2) $\log_3 9 = 2$     |
| 3) $\log_6 1/6 = -1$ | 4) $\log_4 8 = 3/2$   |
| 5) $\log 10000 = 4$  | 6) $\ln 4 = 1.386$    |
| 7) $\log_7 1 = 0$    | 8) $\log_2 64 = 6$    |
| 9) $\log_3 3 = 1$    | 10) $\log_8 16 = 4/3$ |

### **III. Solve for x: (hint: rewrite into exponential format.)**

Example:  $\log_x 36 = 2$  becomes  $x^2 = 36$ , so  $x = 6$ .

- |                    |                      |
|--------------------|----------------------|
| 1) $\log_2 x = 5$  | 2) $\log_x 25 = 2$   |
| 3) $\log_4 16 = x$ | 4) $\log_3 x = 2$    |
| 5) $\log_x 7 = 1$  | 6) $\log_{14} 1 = x$ |
| 7) $\log_8 64 = x$ | 8) $\log_5 5 = x$    |
| 9) $\log_x 2 = .5$ | 10) $\log x = 3$     |

Three properties of logarithms are:

$$\log(xy) = \log x + \log y$$

$$\log(x/y) = \log x - \log y$$

$$\log x^n = n \log x$$

### **IV. Expand these logarithms:**

Example:  $\log(x^3 y^2) = \log x^3 + \log y^2 = 3 \log x + 2 \log y$ .

- |                    |                 |
|--------------------|-----------------|
| 1) $\ln(x^2 y)$    | 2) $\log(10xy)$ |
| 3) $\log(x^3 y/z)$ | 4) $\ln(x/y^2)$ |
| 5) $\ln((xy^2))$   |                 |

### **V. Contract these logarithms:**

Example:  $3 \log x - 2 \log y = \log x^3 - \log y^2 = \log(x^3/y^2)$

- |                                 |                                 |
|---------------------------------|---------------------------------|
| 1) $\log 4 - \log x + 2 \log y$ | 2) $\log x + 2 \log y - \log z$ |
| 3) $\log(x+1) + \log x$         | 4) $\ln x + 2 \ln y + 3 \ln z$  |

**Using logs to solve exponential equations.**

Example:  $7^x = 17$

$\log 7^x = \log 17$

$x \log 7 = \log 17$  (Rule 3)

$x = (\log 17)/(\log 7)$

$x = 1.4559$

Check:  $7^{1.4559} = 17$ .

**VI. Solve for x:**

- |                         |   |
|-------------------------|---|
| 1) $5^x = 10$           | 2) $4^x = 18$   |
| 3) $2^x = 15$           | 4) $3^{(x+1)} = 15$                                       |
| 5) $e^{(2+x)} = 6$      | 6) $x = \log_3 11$ (rewrite into exponential form first!) |
| 7) $x = \log_6 20$      | 8) $x = \log_2 512$                                       |
| 9) $x = \ln 5$          | 10) $4.5^x = 7.9$   |
| 11) $23e^{0.023x} = 76$ | 12) $1500e^{1.24x} = 1200$                                |

**Answers!**

***I.***

- |                    |                        |                         |                     |
|--------------------|------------------------|-------------------------|---------------------|
| 1) $\log_4 64 = 3$ | 2) $\log_2 128 = 7$    | 3) $\log_3 3 = 1$       | 4) $\log_5 1 = 0$   |
| 5) $\log_8 64 = 2$ | 6) $\log_3 (1/3) = -1$ | 7) $\log_4 (1/16) = -2$ | 8) $\log 10000 = 5$ |
| 9) $\log_3 27 = 3$ | 10) $\ln 148.413 = 5$  |                         |                     |
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***II.***

- |                   |                    |                   |                  |
|-------------------|--------------------|-------------------|------------------|
| 1) $2^3 = 8$      | 2) $3^2 = 9$       | 3) $6^{-1} = 1/6$ | 4) $4^{3/2} = 8$ |
| 5) $10^4 = 10000$ | 6) $e^{1.386} = 4$ | 7) $7^0 = 1$      | 8) $2^6 = 64$    |
| 9) $3^1 = 3$      | 10) $8^{4/3} = 16$ |                   |                  |
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***III.***

- |       |          |      |      |
|-------|----------|------|------|
| 1) 32 | 2) 5     | 3) 2 | 4) 9 |
| 5) 7  | 6) 0     | 7) 2 | 8) 1 |
| 9) 4  | 10) 1000 |      |      |
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***IV.***

- |                      |                          |                                 |
|----------------------|--------------------------|---------------------------------|
| 1) $2 \ln x + \ln y$ | 2) $1 + \log x + \log y$ | 3) $3 \log x + \log y - \log z$ |
| 4) $\ln x - 2 \ln y$ | 5) $\ln x + 2 \ln y$     |                                 |
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***V.***

- |                    |                    |                     |                    |
|--------------------|--------------------|---------------------|--------------------|
| 1) $\log (4y^2/x)$ | 2) $\log (xy^2/z)$ | 3) $\log (x^2 + x)$ | 4) $\ln (xy^2z^3)$ |
|--------------------|--------------------|---------------------|--------------------|
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***VI.***

- |          |           |            |           |
|----------|-----------|------------|-----------|
| 1) 1.431 | 2) 2.085  | 3) 3.908   | 4) 1.465  |
| 5) -.208 | 6) 2.183  | 7) 1.672   | 8) 9      |
| 9) 1.609 | 10) 1.374 | 11) 51.967 | 12) -0.18 |