

## Factoring Trinomials

Trinomials are polynomials with three terms. If the highest power is 2, they are also called quadratics. It is most important to be able to factor trinomials of the form

$$ax^2 + bx + c$$

Trinomials with  $a = 1$  of the form  $x^2 + bx + c$  can usually be factored easily if  $b$  and  $c$  are small.

If the second sign is positive, the two signs will be the same.

$$x^2 + bx + c = (x + \quad)(x + \quad) \quad \text{or} \quad x^2 - bx + c = (x - \quad)(x - \quad)$$

Notice that the middle term tells us whether both of the signs are positive or both are negative.

The numbers needed to fill in the blanks:

- 1) should multiply to give  $c$
- 2) should add to give  $b$

Example:

$$x^2 + 5x + 6 = (x + \quad)(x + \quad)$$

To fill the two blanks, we need two numbers that multiply to give 6.  $6 = 1 \times 6$  or  $2 \times 3$

Since the numbers must add to give 5, we use 2 and 3 because  $2 + 3 = 5$ .

$$\text{So, } x^2 + 5x + 6 = (x + 2)(x + 3)$$

If the second sign is negative, the two signs will be different.

$$x^2 + bx - c = (x + \quad)(x - \quad) \quad \text{or} \quad x^2 - bx - c = (x - \quad)(x + \quad)$$

The numbers needed to fill in the blanks:

- 1) should multiply to give  $c$
- 2) should subtract to give  $b$

The middle sign gives the sign of the "larger" number (the number with the larger absolute value).

Example:

$$x^2 - 4x - 12 = (x + \quad)(x - \quad)$$

To fill the two blanks, we need two numbers that multiply to give 12.

$$12 = 1 \times 12 \text{ or } 2 \times 6 \text{ or } 3 \times 4$$

Since the numbers must subtract to give 4, we use 6 and 2 because  $6 - 2 = 4$ . Since the middle sign is negative, the negative sign must go with the larger number, 6. So,

$$x^2 - 4x - 12 = (x + 2)(x - 6)$$

## AC Method

(also called by some authors the By Grouping, FOIL, or Master Product)  
for factoring trinomials of the form  
 $ax^2 + bx + c$

If coefficient of the first term is not 1, the polynomial may be complicated to factor. The method we will most use is called the "Master Product" method or "a-c" method.

Example: Consider  $5x^2 + 8x - 4$

Step 1: Multiply **a** and **c** together. This number is called the master product.

Example:  $5 \cdot (-4) = -20$

Step 2: List all pairs of numbers that multiply to give you the master product.

Example:  $-20 = 1 \cdot (-20)$   
 $(-1) \cdot 20$   
 $2 \cdot (-10)$   
 $(-2) \cdot 10$   
 $4 \cdot (-5)$   
 $(-4) \cdot 5$

Step 3: Examine the pairs to find one that adds to give the middle number, **b**.

Example:  $-2 + 10 = 8$   
so the pair is  $-2$  and  $10$

Step 4: Write the middle term as a sum of two terms having coefficients of the pair of numbers found in Step 3. You should now have four terms.

Example:  $5x^2 + 8x - 4 = 5x^2 - 2x + 10x - 4$

Step 5: Factor by grouping. Group the first two terms together and the last two terms together using parentheses. Remove any common factors from each group. What remains in the two sets of parentheses should be the same. This will give one factor. What is left will give the other factor.

Example:  $5x^2 - 2x + 10x - 4$   
 $(5x^2 - 2x) + (10x - 4)$   
 $x(5x - 2) + 2(5x - 2)$   
 $(5x - 2)(x + 2)$