Integration Using Substitution

$$\int \left(x^2+2x+5\right)^5(2x+2)\,dx$$

$Let:$

$$u = x^2+2x+5$$
$$du = (2x+2)\,dx$$

$$\begin{aligned}\int \left(x^2+2x+5\right)^5(2x+2)\,dx &= \int u^5\,du\\ &= \frac{u^6}{6}+c\\ &= \frac{1}{6}\left(x^2+2x+5\right)^6+c\end{aligned}$$

$$\int 2t\left(e^{t^2}\right)dt$$

$let:$

$$u = t^2$$
$$du = (2t)\,dt$$

$$\begin{aligned}\int 2t\left(e^{t^2}\right)dt &= \int e^u\,du\\ &= e^u+c\\ &= e^{t^2}+c\end{aligned}$$

$$\int 3(3x+4)^6\,dx$$

$Let:$

$$u = 3x+4$$
$$du = 3dx$$

$$\begin{aligned}\int 3(3x+4)^6\,dx &= \int (3x+4)^6(3dx)\\ &= \int u^6\,du\\ &= \frac{1}{7}u^7+c\\ &= \frac{1}{7}(3x+4)^7+c\end{aligned}$$

$$\int te^{-t^2}\,dt$$

$let:$

$$u = -t^2$$
$$du = (-2t)\,dt$$

$$\begin{aligned}\int te^{-t^2}\,dt &= \int te^{-t^2}\left(\frac{-2}{-2}\right)dt\\ * \quad &= -\frac{1}{2}\int e^{-t^2}(-2t)\,dt\\ &= -\frac{1}{2}\int e^u\,du\\ &= -\frac{1}{2}e^u+c\\ &= -\frac{1}{2}e^{-t^2}+c\end{aligned}$$

$$\int te^{-t^2}\,dt$$

$let:$

$$u = -t^2$$
$$du = (-2t)\,dt$$
$$\frac{du}{-2} = tdt$$

$$\begin{aligned}* \int te^{-t^2}\,dt &= \int e^u\,\frac{1}{-2}\,du\\ &= -\frac{1}{2}\int e^u\,du\\ &= -\frac{1}{2}e^u+c\\ &= -\frac{1}{2}e^{-t^2}+c\end{aligned}$$

$$\int \frac{1}{4x+7}\,dt$$

$let:$

$$u = 4x+7$$
$$du = 4dt$$

$$\begin{aligned}\int \frac{1}{4x+7}\,dt &= \int \frac{1}{4x+7}\left(\frac{4}{4}\right)dt\\ &= \frac{1}{4}\int \frac{1}{4x+7}(4dt)\\ &= \frac{1}{4}\int \frac{1}{u}\,du\\ &= \frac{1}{4}\ln|u|+c\\ &= \frac{1}{4}\ln|4x+7|+c\end{aligned}$$

$$\int 4x^2\left(x^3+5\right)^3dx$$

$let:$

$$u = x^3+5$$
$$du = 3x^2dx$$

$$\begin{aligned}\int 4x^2\left(x^3+5\right)^3dx &= 4\int x^2\left(x^3+5\right)^3dx\\ &= 4\int \left(x^3+5\right)^3\left(\frac{3}{3}\right)\left(x^2\right)dx\\ &= \frac{4}{3}\int \left(x^3+5\right)^3\left(3x^2\right)dx\\ &= \frac{4}{3}\int u^3\,du\\ &= \left(\frac{4}{3}\right)\left(\frac{u^4}{4}\right)+c = \frac{u^4}{3}+c\\ &= \frac{\left(x^3+5\right)^4}{3}+c\end{aligned}$$

* denotes the same problem worked in different ways

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