Integration by Parts

Integration by parts is a reversal of the product rule for differentiation. It is used when the integrand is a product of two functions, and no substitution can be found.

Step 1: Selection of parts-
The integrand should be broken up into 2 parts. One part will be named u, the other part, which contains dx, will be called dv. The product of $u\ dv$ must equal the original integrand. It must be possible to integrate dv, preferably using standard formulas or simple substitutions. The integral of dv will be called v. The derivative of u multiplied by dx will be called du. The product of $v\ du$ should be no more complicated to integrate than $u\ dv$.

Step 2: Find du by taking the derivative of u and multiplying by dx. Find v by taking the integral of dv. It often helps to fill out a chart like the one below:

$u =$	$dv =$
$du =$	$v =$

Step 3: The formula for integration by parts is as follows:

$$\int u\ dv = uv - \int v\ du$$

Substitute u, v, du and dv into the right side of the formula.

Note: it is still necessary to integrate $\int v\ du$.

Example: $\int xe^x dx$

Let $u = x$ and let $dv = e^x$

$u = x$	$dv = e^x$
$du = 1 \cdot dx$	$v = e^x$

$$\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x + C\ .$$

Check by differentiation: $\frac{d}{dx}(xe^x) = x\ e^x + e^x - e^x = xe^x$

Tips for selecting *u* and *dv*:

One possible choice of u and dv is to let $dv = dx$ and the rest of the integrand be u.

If the integrand is of the form $x^p e^{ax}$, try using $u = x^p$ and $dv = e^{ax}$.

If the integrand is of the form $x^p (\ln x)^q$, try using $u = x^p$ and $dv = (\ln x)^q$.

If the integrand $\int v\ du$ is the same as the original, solve the formula for integration by parts algebraically to find the unknown integral.

If the integral $\int v\ du$ still cannot be solved, try a different selection of parts.

You may need to use integration by parts several times in one problem.