

Logarithms

For $x > 0$, $a > 0$, and $a \neq 1$

$$x = \log_b a$$

logarithmic form

means

$$b^x = a$$

exponential form

Properties of Logarithms

1. $\log_b 1 = 0$
2. $\log_b b = 1$
3. $\log_b b^x = x$
4. $\log_b x = \log_b y$ if and only if $x = y$
5. $\log_b (uv) = \log_b u + \log_b v$
6. $\log_b \left(\frac{u}{v}\right) = \log_b u - \log_b v$
7. $\log_b u^n = n \log_b u$

Change of Base Formula

For any base k , $\log_b a = \frac{\log_k a}{\log_k b}$

Common Errors

$$\log_b u - \log_b v \neq \frac{\log_b u}{\log_b v}$$

$$\log_b u + \log_b v \neq (\log_b u)(\log_b v)$$

There is no formula to simplify these expressions:

$$\log_b (u + v) \text{ and } \log_b (u - v)$$

Solving Exponential Equations -

An exponential equation is one with the variable in an exponent.

1. Isolate the exponential expression (the base and exponent).

$$\begin{aligned} \text{Ex: } 100 \times 1.02^{3x} &= 200 && \text{Divide both sides by 100} \\ 1.02^{3x} &= 2 \end{aligned}$$

2. Take the logarithm (log or ln) of both sides and bring down the exponent using property 7 on the other side of this page.

$$\begin{aligned} \text{Ex: } \ln 1.02^{3x} &= \ln 2 \\ 3x \ln 1.02 &= \ln 2 \end{aligned}$$

3. Solve for the variable and evaluate using a calculator if necessary.

$$\text{Ex: } x = \frac{\ln 2}{2 \ln 1.02} \approx 11.67$$

Solving Logarithmic Equations

1. Move all terms containing logarithms to one side of the equation, and all other terms to the other side of the equation.

$$\begin{aligned} \text{Ex: } 2 \log_3 x + 3 &= \log_3 4 + 5 \\ 2 \log_3 x - \log_3 4 &= 5 - 3 \end{aligned}$$

2. Combine the terms with logarithms to get a single logarithm with a coefficient of 1 using properties 5, 6, and 7 (work from the right side to the left of each property).

$$\begin{aligned} \text{Ex: } \log_3 x^2 - \log_3 4 &= 5 - 3 && \text{Property 7} \\ \log_3 \frac{x^2}{4} &= 2 && \text{Property 6} \end{aligned}$$

3. Rewrite the equation in exponential form.

$$\text{Ex: } 3^2 = \frac{x^2}{4}$$

4. Solve for the variable and check all solutions in the original equation.

$$\begin{aligned} \text{Ex: } 9 &= \frac{x^2}{4} \\ 36 &= x^2 \\ \pm 6 &= x \end{aligned}$$

When checking, -6 does not give a solution since the domain of all logarithmic functions is $x > 0$. Therefore, the solution is $x = 6$.

NOTE: These procedures work for most equations, but additional techniques such as factoring may be required. Some exponential and logarithmic equations cannot be solved.