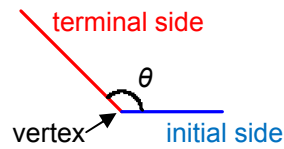


Trig Terms and Concepts That You Need to Memorize Chapter 1

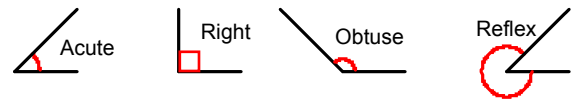
An **angle** θ in a plane consists of two rays with a common endpoint. You can think of an angle having an initial ray that remains fixed and another ray that rotates about the common endpoint. The common endpoint is called the **vertex** of the angle, the fixed ray as the **initial side** of the angle, and the rotating ray as the **terminal side** of the angle.



Angles can be measured in degrees, where a counterclockwise rotation is positive and a clockwise rotation is negative. Degrees are subdivided into minutes and minutes are subdivided into seconds. There are 60 minutes ($60'$) in 1 degree (1°) and 60 seconds ($60''$) in 1 minutes ($1'$). There are 360° in one complete revolution

An angle is classified by its measure:

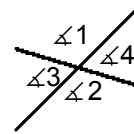
- An **acute angle** has measure between 0° and 90° .
- A **right angle** has measure 90° .
- An **obtuse angle** has measure between 90° and 180° .
- A **reflex angle** has measure between 180° and 360° .



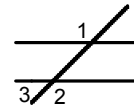
Two angles are

- **Complementary** if the sum of their measures is 90° .
- **Supplementary** if the sum of their measures is 180°

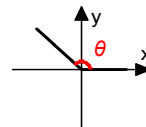
Vertical angles are formed by two intersecting lines and vertical angles are equal. In the figure, $\sphericalangle 1$ and $\sphericalangle 2$ are vertical angles, and $\sphericalangle 3$ and $\sphericalangle 4$ are vertical angles. For example, $\sphericalangle 1$ is supplementary to $\sphericalangle 3$ and supplementary to $\sphericalangle 4$.



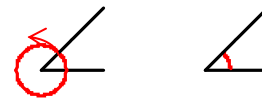
When two parallel lines are cut by a transversal, any pair of angles are either equal or supplementary. For example, $\sphericalangle 1$ is equal to $\sphericalangle 2$ and supplementary to $\sphericalangle 3$.



An angle is in **standard position** if the initial side lies on the positive x-axis and the vertex is at the origin.



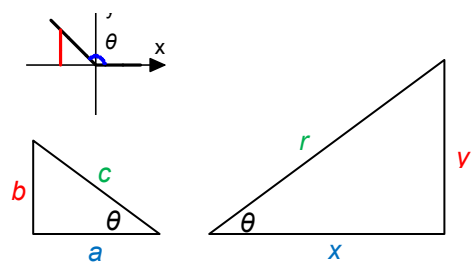
Coterminal angles are angles drawn in standard position that have the same initial and terminal sides. The measures of coterminal angles differ by an integer multiple of 360° .



For any angle drawn in standard position, a **reference triangle** is formed by drawing a vertical line segment from any point on the terminal side of an angle to the x-axis.

Similar Triangle Theorem The ratio of any two sides of one triangle is the same as the ratio of the corresponding sides of any other similar triangle. In the two similar triangles shown,

$$\frac{a}{c} = \frac{x}{r} \quad \frac{b}{c} = \frac{y}{r} \quad \frac{b}{a} = \frac{y}{x}$$



We **define six trig functions** where the ratio of two sides of a triangle associated with an angle drawn in standard position is a function of the angle the terminal side of the angle makes with the positive x-axis. If θ is an angle in standard position and the point (x, y) is any point on the terminal side of θ , the six trig functions are defined as:

Sine

$$\sin \theta = \frac{y}{r}$$

Cosine

$$\cos \theta = \frac{x}{r}$$

Tangent

$$\tan \theta = \frac{y}{x}$$

Cotangent

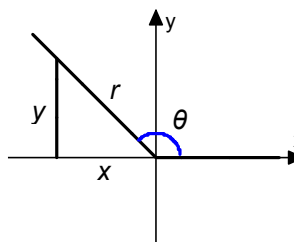
$$\cot \theta = \frac{x}{y}$$

Secant

$$\sec \theta = \frac{r}{x}$$

Cosecant

$$\csc \theta = \frac{r}{y}$$



Given any two values of x , y , and r , the missing value can be found by using the Pythagorean Theorem:

$$x^2 + y^2 = r^2$$

An alternate way to define the six trig functions is by using a right triangle and the relationship of the sides to the angle θ , where *opp* represents the side opposite θ , *adj* represents the side adjacent to θ , and *hyp* represents the hypotenuse.

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

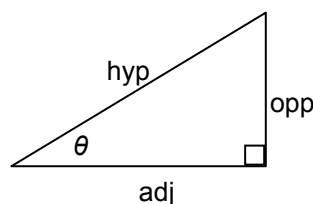
$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\cot \theta = \frac{\text{adj}}{\text{opp}}$$

$$\sec \theta = \frac{\text{hyp}}{\text{adj}}$$

$$\csc \theta = \frac{\text{hyp}}{\text{opp}}$$



The mnemonic often used to remember these relationships is SOH CAH TOA.

The angles 0° , 30° , 45° , and 60° occur frequently in trig applications and they are called **special angles**. Angles that lie on the axes (90° , 180° , 270° , and 360°) also occur frequently and they are called **quadrantal angles**.

Trig Function of Important Angles

θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
45°	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined
180°	0	-1	0
270°	-1	0	undefined
360°	0	1	0

Fundamental Identities An *identity* is an equation that is true for all possible values of the variables. Identities are used to rewrite an expression in another form.

Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

Ratio Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$