

Solving More TRIG EQUATIONS

In Problems 7–30, solve each equation on the interval $0 \leq \theta < 2\pi$.

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| 7. $2 \sin \theta + 3 = 2$ | 8. $1 - \cos \theta = \frac{1}{2}$ | 9. $4 \cos^2 \theta = 1$ | 10. $\tan^2 \theta = \frac{1}{3}$ |
| 11. $2 \sin^2 \theta - 1 = 0$ | 12. $4 \cos^2 \theta - 3 = 0$ | 13. $\sin(3\theta) = -1$ | 14. $\tan \frac{\theta}{2} = \sqrt{3}$ |
| 15. $\cos(2\theta) = -\frac{1}{2}$ | 16. $\tan(2\theta) = -1$ | 17. $\sec \frac{3\theta}{2} = -2$ | 18. $\cot \frac{2\theta}{3} = -\sqrt{3}$ |
| 19. $2 \sin \theta + 1 = 0$ | 20. $\cos \theta + 1 = 0$ | 21. $\tan \theta + 1 = 0$ | 22. $\sqrt{3} \cot \theta + 1 = 0$ |
| 23. $4 \sec \theta + 6 = -2$ | 24. $5 \csc \theta - 3 = 2$ | 25. $3\sqrt{2} \cos \theta + 2 = -1$ | 26. $4 \sin \theta + 3\sqrt{3} = \sqrt{3}$ |
| 27. $\cos\left(2\theta - \frac{\pi}{2}\right) = -1$ | 28. $\sin\left(3\theta + \frac{\pi}{18}\right) = 1$ | 29. $\tan\left(\frac{\theta}{2} + \frac{\pi}{3}\right) = 1$ | 30. $\cos\left(\frac{\theta}{3} - \frac{\pi}{4}\right) = \frac{1}{2}$ |

In Problems 31–40, solve each equation. Give a general formula for all the solutions. List six solutions.

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| 31. $\sin \theta = \frac{1}{2}$ | 32. $\tan \theta = 1$ | 33. $\tan \theta = -\frac{\sqrt{3}}{3}$ | 34. $\cos \theta = -\frac{\sqrt{3}}{2}$ | 35. $\cos \theta = 0$ |
| 36. $\sin \theta = \frac{\sqrt{2}}{2}$ | 37. $\cos(2\theta) = -\frac{1}{2}$ | 38. $\sin(2\theta) = -1$ | 39. $\sin \frac{\theta}{2} = -\frac{\sqrt{3}}{2}$ | 40. $\tan \frac{\theta}{2} = -1$ |

In Problems 41–52, use a calculator to solve each equation on the interval $0 \leq \theta < 2\pi$. Round answers to two decimal places.

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| 41. $\sin \theta = 0.4$ | 42. $\cos \theta = 0.6$ | 43. $\tan \theta = 5$ | 44. $\cot \theta = 2$ |
| 45. $\cos \theta = -0.9$ | 46. $\sin \theta = -0.2$ | 47. $\sec \theta = -4$ | 48. $\csc \theta = -3$ |
| 49. $5 \tan \theta + 9 = 0$ | 50. $4 \cot \theta = -5$ | 51. $3 \sin \theta - 2 = 0$ | 52. $4 \cos \theta + 3 = 0$ |

PART II

In Problems 5–46, solve each equation on the interval $0 \leq \theta < 2\pi$.

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| 5. $2 \cos^2 \theta + \cos \theta = 0$ | 6. $\sin^2 \theta - 1 = 0$ | 7. $2 \sin^2 \theta - \sin \theta - 1 = 0$ |
| 8. $2 \cos^2 \theta + \cos \theta - 1 = 0$ | 9. $(\tan \theta - 1)(\sec \theta - 1) = 0$ | 10. $(\cot \theta + 1)\left(\csc \theta - \frac{1}{2}\right) = 0$ |
| 11. $\sin^2 \theta - \cos^2 \theta = 1 + \cos \theta$ | 12. $\cos^2 \theta - \sin^2 \theta + \sin \theta = 0$ | 13. $\sin^2 \theta = 6(\cos \theta + 1)$ |
| 14. $2 \sin^2 \theta = 3(1 - \cos \theta)$ | 15. $\cos(2\theta) + 6 \sin^2 \theta = 4$ | 16. $\cos(2\theta) = 2 - 2 \sin^2 \theta$ |
| 17. $\cos \theta = \sin \theta$ | 18. $\cos \theta + \sin \theta = 0$ | 19. $\tan \theta = 2 \sin \theta$ |
| 20. $\sin(2\theta) = \cos \theta$ | 21. $\sin \theta = \csc \theta$ | 22. $\tan \theta = \cot \theta$ |
| 23. $\cos(2\theta) = \cos \theta$ | 24. $\sin(2\theta) \sin \theta = \cos \theta$ | 25. $\sin(2\theta) + \sin(4\theta) = 0$ |
| 26. $\cos(2\theta) + \cos(4\theta) = 0$ | 27. $\cos(4\theta) - \cos(6\theta) = 0$ | 28. $\sin(4\theta) - \sin(6\theta) = 0$ |
| 29. $1 + \sin \theta = 2 \cos^2 \theta$ | 30. $\sin^2 \theta = 2 \cos \theta + 2$ | 31. $2 \sin^2 \theta - 5 \sin \theta + 3 = 0$ |
| 32. $2 \cos^2 \theta - 7 \cos \theta - 4 = 0$ | 33. $3(1 - \cos \theta) = \sin^2 \theta$ | 34. $4(1 + \sin \theta) = \cos^2 \theta$ |
| 35. $\tan^2 \theta = \frac{3}{2} \sec \theta$ | 36. $\csc^2 \theta = \cot \theta + 1$ | 37. $3 - \sin \theta = \cos(2\theta)$ |
| 38. $\cos(2\theta) + 5 \cos \theta + 3 = 0$ | 39. $\sec^2 \theta + \tan \theta = 0$ | 40. $\sec \theta = \tan \theta + \cot \theta$ |
| 41. $\sin \theta - \sqrt{3} \cos \theta = 1$ | 42. $\sqrt{3} \sin \theta + \cos \theta = 1$ | 43. $\tan(2\theta) + 2 \sin \theta = 0$ |
| 44. $\tan(2\theta) + 2 \cos \theta = 0$ | 45. $\sin \theta + \cos \theta = \sqrt{2}$ | 46. $\sin \theta + \cos \theta = -\sqrt{2}$ |

In Problems 47–52, find the x -intercepts of the graph of each trigonometric function on the interval $0 \leq x < 2\pi$.

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| 47. $f(x) = 4 \cos^2 x - 1$ | 48. $f(x) = 4 \sin^2 x - 3$ | 49. $f(x) = \sin(2x) - \sin x$ |
| 50. $f(x) = \cos(2x) + \cos x$ | 51. $f(x) = \sin^2 x + 2 \cos x + 2$ | 52. $f(x) = \cos(2x) + \sin^2 x$ |

In Problems 53–64, use a graphing utility to solve each equation. Express the solution(s) rounded to two decimal places.

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| 53. $x + 5 \cos x = 0$ | 54. $x - 4 \sin x = 0$ | 55. $22x - 17 \sin x = 3$ |
| 56. $19x + 8 \cos x = 2$ | 57. $\sin x + \cos x = x$ | 58. $\sin x - \cos x = x$ |
| 59. $x^2 - 2 \cos x = 0$ | 60. $x^2 + 3 \sin x = 0$ | 61. $x^2 - 2 \sin(2x) = 3x$ |
| 62. $x^2 = x + 3 \cos(2x)$ | 63. $6 \sin x - e^x = 2, \quad x > 0$ | 64. $4 \cos(3x) - e^x = 1, \quad x > 0$ |

Applications and Extensions

53. What are the x -intercepts of the graph of $f(x) = 4 \sin^2 x - 3$ on the interval $[0, 2\pi]$?
54. What are the x -intercepts of the graph of $f(x) = 2 \cos(3x) + 1$ on the interval $[0, \pi]$?
55. $f(x) = 3 \sin x$.
- Find the x -intercepts of the graph of f on the interval $[-2\pi, 4\pi]$.
 - Graph $f(x) = 3 \sin x$ on the interval $[-2\pi, 4\pi]$.
 - Solve $f(x) = \frac{3}{2}$ on the interval $[-2\pi, 4\pi]$. What points are on the graph of f ? Label these points on the graph drawn in part (b).
 - Use the graph drawn in part (b) along with the results of part (c) to determine the values of x such that $f(x) > \frac{3}{2}$ on the interval $[-2\pi, 4\pi]$.
56. $f(x) = 2 \cos x$.
- Find the x -intercepts of the graph of f on the interval $[-2\pi, 4\pi]$.
 - Graph $f(x) = 2 \cos x$ on the interval $[-2\pi, 4\pi]$.
 - Solve $f(x) = -\sqrt{3}$ on the interval $[-2\pi, 4\pi]$. What points are on the graph of f ? Label these points on the graph drawn in part (b).
 - Use the graph drawn in part (b) along with the results of part (c) to determine the values of x such that $f(x) < -\sqrt{3}$ on the interval $[-2\pi, 4\pi]$.
57. $f(x) = 4 \tan x$.
- Solve $f(x) = -4$.
 - For what values of x is $f(x) < -4$ on the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$?
58. $f(x) = \cot x$.
- Solve $f(x) = -\sqrt{3}$.
 - For what values of x is $f(x) > -\sqrt{3}$ on the interval $(0, \pi)$?
59. (a) Graph $f(x) = 3 \sin(2x) + 2$ and $g(x) = \frac{7}{2}$ on the same Cartesian plane for the interval $[0, \pi]$.
- Solve $f(x) = g(x)$ on the interval $[0, \pi]$ and label the points of intersection on the graph drawn in part (b).
 - Solve $f(x) > g(x)$ on the interval $[0, \pi]$.
 - Shade the region bounded by $f(x) = 3 \sin(2x) + 2$ and $g(x) = \frac{7}{2}$ between the two points found in part (b) on the graph drawn in part (a).

60. (a) Graph $f(x) = 2 \cos \frac{x}{2} + 3$ and $g(x) = 4$ on the same Cartesian plane for the interval $[0, 4\pi]$.

66. Projectile Motion An object is propelled upward at an angle θ , $45^\circ < \theta < 90^\circ$, to the horizontal with an initial velocity of v_0 feet per second from the base of an inclined plane that makes an angle of 45° with the horizontal. See the illustration. If air resistance is ignored, the distance R that the object travels up the inclined plane is given by

$$R(\theta) = \frac{v_0^2 \sqrt{2}}{32} [\sin(2\theta) - \cos(2\theta) - 1]$$

- (a) In calculus, you will be asked to find the angle θ that maximizes R by solving the equation $\sin(2\theta) + \cos(2\theta) = 0$. Solve this equation for θ using the method of Example 7.
- (b) Solve this equation for θ by dividing each side by $\cos(2\theta)$.

- Solve $f(x) = g(x)$ on the interval $[0, 4\pi]$ and label the points of intersection on the graph drawn in part (b).
 - Solve $f(x) < g(x)$ on the interval $[0, 4\pi]$.
 - Shade the region bounded by $f(x) = 2 \cos \frac{x}{2} + 3$ and $g(x) = 4$ between the two points found in part (b) on the graph drawn in part (a).
61. (a) Graph $f(x) = -4 \cos x$ and $g(x) = 2 \cos x + 3$ on the same Cartesian plane for the interval $[0, 2\pi]$.
- Solve $f(x) = g(x)$ on the interval $[0, 2\pi]$ and label the points of intersection on the graph drawn in part (b).
 - Solve $f(x) > g(x)$ on the interval $[0, 2\pi]$.
 - Shade the region bounded by $f(x) = -4 \cos x$ and $g(x) = 2 \cos x + 3$ between the two points found in part (b) on the graph drawn in part (a).
62. (a) Graph $f(x) = 2 \sin x$ and $g(x) = -2 \sin x + 2$ on the same Cartesian plane for the interval $[0, 2\pi]$.
- Solve $f(x) = g(x)$ on the interval $[0, 2\pi]$ and label the points of intersection on the graph drawn in part (b).
 - Solve $f(x) > g(x)$ on the interval $[0, 2\pi]$.
 - Shade the region bounded by $f(x) = 2 \sin x$ and $g(x) = -2 \sin x + 2$ between the two points found in part (b) on the graph drawn in part (a).

63. The Ferris Wheel In 1893, George Ferris engineered the Ferris Wheel. It was 250 feet in diameter. If the wheel makes 1 revolution every 40 seconds, then the function

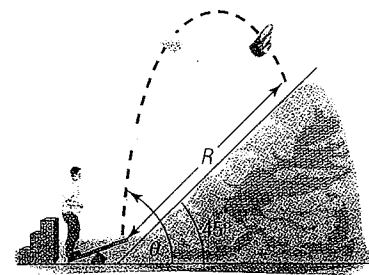
$$h(t) = 125 \sin\left(0.157t - \frac{\pi}{2}\right) + 125$$

represents the height h , in feet, of a seat on the wheel as a function of time t , where t is measured in seconds. The ride begins when $t = 0$.

- During the first 40 seconds of the ride, at what time t is an individual on the Ferris Wheel exactly 125 feet above the ground?
- During the first 80 seconds of the ride, at what time t is an individual on the Ferris Wheel exactly 250 feet above the ground?
- During the first 40 seconds of the ride, over what interval of time t is an individual on the Ferris Wheel more than 125 feet above the ground?

64. Tire Rotation The P215/65R15 Cobra Radial G/T tire has a diameter of exactly 26 inches. Suppose that a car's wheel is making 2 revolutions per second (the car is traveling a little less than 10 miles per hour). Then $h(t) = 13 \sin\left(4\pi t - \frac{\pi}{2}\right) + 13$ represents the height h (in inches) of a point on the tire as a function of time t (in seconds).

STARTS TO MOVE when $t = 0$.



- What is the maximum distance R if $v_0 = 32$ feet per second?
- Graph $R = R(\theta)$, $45^\circ \leq \theta \leq 90^\circ$, and find the angle θ that maximizes the distance R . Also find the maximum distance. Use $v_0 = 32$ feet per second. Compare the results with the answers found earlier.