Answers for Review Problems for Calculus 1

1. $x \in \mathbb{R}$, $y \geq -\frac{4}{3}$

$$3x^2 + 2x - 1 = 0$$
$$(3x-1)(x+1) = 0$$
$$3x - 1 = 0 \quad x + 1 = 0$$
$$x = \frac{1}{3} \quad or \quad x = -1$$

2. $x \in \mathbb{R}$, $y \in \mathbb{R}$

$$12t^3 + 12t^2 - 12t = 0$$
$$t(t^2 + t - 1) = 0$$
$$t = 0 \ or \ t^2 + t - 1 = 0$$
$$t = \frac{-1 \pm \sqrt{5}}{2}$$

3. $x \in \mathbb{R}$, $-.125 \leq y \leq 1$

$$\frac{\left(r^2+1\right) - 2r^2}{(r^2+1)^2} = 0$$
$$r^2 - 2r^2 + 1 = 0$$
$$-r^2 = -1$$
$$r = \pm 1$$

4. $x \in \mathbb{R}$, $-0.409 < y < 1.471$

$$\frac{x^2 + x + 1 - (x+1)(2x+1)}{\left(x^2 + x + 1\right)^2} = 0$$
$$x^2 + x + 1 - 2x^2 - 3x - 1 = 0$$
$$-x^2 - 2x = 0$$
$$-x(x+2) = 0$$
$$x = 0 \ or \ x = -2$$

5. $x \neq 0$, $y > -4.75$ or $y < -21.8$

$$\frac{(x-4)(14x-16)}{5x^{\frac{1}{5}}} = 0$$
$$x - 4 = 0 \quad or \quad 14x - 16 = 0$$
$$x = 4 \qquad or \qquad x = \frac{8}{7}$$

6. $x \neq 0, x \neq 1$, $y \in \mathbb{R}$

$$\frac{1}{3}\left(x^2 - x\right)^{-\frac{2}{3}}(2x-1) = 0$$
$$2x - 1 = 0$$
$$x = \frac{1}{2}$$

7. $x \in \mathbb{R}$, $-2 \leq y \leq 2$

$$2\sin(2\theta)\cos(2\theta)2 = 0$$
$$\sin 2\theta = 0 \ or \ \cos 2\theta = 0$$
For $k \in \mathbb{Z}$
$$2\theta_1 = 0 + 2k\pi \quad or \quad 2\theta_3 = \frac{\pi}{2} + 2k\pi$$
$$2\theta_2 = \pi + 2k\pi \ \ or \ \ 2\theta_4 = \frac{3\pi}{2} + 2k\pi$$
$$\theta_1 = k\pi$$
$$\theta_2 = \frac{\pi}{2} + k\pi$$
$$\theta_3 = \frac{\pi}{4} + k\pi$$
$$\theta_4 = \frac{3\pi}{4} + k\pi$$

8. $x > 0$, $y \in \mathbb{R}$

$$x\left(\frac{1}{x}\right) + \ln x = 0$$
$$\ln x = -1$$
$$x = e^{-1} \approx 0.3679$$

9. $x \in \mathbb{R}$, y > –0.135

$$x\left(2e^{2x}\right) + e^{2x} = 0$$
$$e^{2x}(2x+1) = 0$$
$$2x + 1 = 0$$
$$x = -\frac{1}{2}$$

10. $-3 \leq x \leq 3$, $y \in \mathbb{R}$, $\frac{-x}{\sqrt{9-x^2}} = 0$
$$x = 0$$

11. $x \neq 0$, $y \in \mathbb{Z}$

$$2x - \frac{2}{x^2} = 0$$
$$2x^3 - 2 = 0$$
$$x^3 - 1 = 0$$
$$x^3 = 1$$
$$x = 1$$

12. $x \in \mathbb{R}$, $y \leq 2$

$$\frac{\left(x^2+4\right) - 2x^2}{\left(x^2+4\right)2} = 0$$
$$x^2 + 4 - 2x^2 = 0$$
$$-x^2 + 4 = 0$$
$$x^2 = 4$$
$$x = \pm 2$$

13. $x \in \mathbb{R}$, $-1.4143 < y < 1.4143$

$$\cos x - \sin x = 0$$
$$\cos x = \sin x$$
$$\tan x = 1$$
$$x = \frac{\pi}{4} + k\pi$$

14. $x \in \mathbb{R}$, $-1 \leq y \leq 3$

$$1 + 2\sin x = 0$$
$$\sin x = -\frac{1}{2}$$

Reference angle $\frac{\pi}{6}$

$$Q3: x = \pi + \frac{\pi}{6} + 2k\pi = \frac{7\pi}{6} + 2k\pi$$
$$Q4: x = 2\pi - \frac{\pi}{6} + 2k\pi = \frac{11\pi}{6} + 2k\pi$$

15. $x \in \mathbb{R}$, $y \leq 1$

$$x\left(e^{-x}\right) + e^{-x} = 0$$
$$e^{-x}(x+1) = 0$$
$$x + 1 = 0$$
$$x = -1$$

16. $x > 0$, $y \in \mathbb{R}$

$$\frac{x\left(\frac{1}{x}\right) - \ln x}{x^2} = 0$$
$$1 - \ln x = 0$$
$$\ln x = 1$$
$$x = e \approx 2.71828$$

17. $x \in \mathbb{R}$, y>–1.08

$$e^{x^3 - x}(3x^2 - 1) = 0$$
$$3x^2 - 1 = 0$$
$$x^2 = \frac{1}{3}$$
$$x = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{3}}{3}$$

18. $0 < x < 1$, $y < 0.59$

$$x\frac{1-2x}{2\sqrt{x-x^2}} + \sqrt{x-x^2} = 0$$
$$x(1-2x) + 2\left(x - x^2\right) = 0$$
$$x - 2x^2 + 2x - 2x^2 = 0$$
$$-4x^2 + 3x = 0$$
$$-x(4x-3) = 0$$
$$\underset{\textit{extraneous}}{x = 0} \quad \textit{or} \quad x = \frac{3}{4}$$

19. $x \in \mathbb{R}$, $-\frac{1}{3} \leq y \leq 1$

$$\frac{(2+\sin x)(-\sin x) - (\cos x)(\cos x)}{(2+\sin x)^2} = 0$$
$$-2\sin x - \sin^2 x - \cos^2 x = 0$$
$$-\sin^2 x - 2\sin x - \left(1 - \sin^2 x\right) = 0$$
$$2\sin x + 1 = 0$$
$$\sin x = -\frac{1}{2}$$

See #14 solution

20. $x \in \mathbb{R}$, $y \notin \mathbb{R}$

$$8 - 8x^7 = 0$$
$$1 - x^7 = 0$$
$$x^7 = 1$$
$$x = 1$$

21. $x \in \mathbb{R}$, $-1 \le y \le 3$,

$1 - 2\cos x = 0$

$\cos x = \frac{1}{2}$

$x = \pm\frac{\pi}{3} + 2k\pi$

22. $x \in \mathbb{R}$, $-2 \le y \le 2$

$2\sin x = 0$

$\sin x = 0$

$x_1 = 0 + 2k\pi$

$x_2 = \pi + 2k\pi$

or

$x=\pi+k\pi$

23. $x \neq -1$, $y \in \mathbb{R}$

$$\frac{(1+x)^2 - 2x(1+x)}{\left[(1+x)^2\right]^2} = 0$$

$1 + 2x + x^2 - 2x - 2x^2 = 0$

$-x^2 + 1 = 0$

$x = 1$ or $x = -1$ *extraneous*

24. $x \neq$ -1, $y <$ 0.008

$$\frac{(1+x)^3(-1) - 3(1-x)(1+x)^2}{\left[(1+x)^3\right]^2} = 0$$

$-\left(1 + 3x + 3x^2 + x^3\right) - 3(1-x)\left(1 + 2x + x^2\right) = 0$

$\left(-1 - 3x - 3x^2 - x^3\right) - 3\left(1 + 2x + x^2 - x - 2x^2 - x^3\right) = 0$

$\left(-1 - 3x - 3x^2 - x^3\right) - 3\left(1 + x - x^2 - x^3\right) = 0$

$-1 - 3x - 3x^2 - x^3 - 3 - 3x + 3x^2 + 3x^3 = 0$

$-4 - 6x + 2x^3 = 0$

From graph of $y = 2x^3 - 6x - 4$: $x = -1$ or $x = 2$ *extraneous*

25. $x \in \mathbb{R}$, $y >$ –0.135

$xe^x + e^x = 0$

$e^x(x+1) = 0$

$x + 1 = 0$

$x = -1$

26. $x \in \mathbb{R}$, $y >$ –0.05

$e^x + (x+1)e^x = 0$

$e^x(1 + x + 1) = 0$

$e^x(x+2) = 0$

$x + 2 = 0$

$x = -2$

27. $x \in \mathbb{R}$, $y >$ –0.462

$x^2e^x + 2xe^x = 0$

$e^x\left(x^2 + 2x\right) = 0$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0$ or $x = -2$

28. $x \in \mathbb{R}$, $y >$ –0.42

$\left(x^2 + 2x\right)e^x + e^x(2x+2) = 0$

$e^x\left(x^2 + 2x + 2x + 2\right) = 0$

$e^x\left(x^2 + 4x + 2\right) = 0$

$x^2 + 4x + 2 = 0$

$$x = \frac{-4 \pm \sqrt{16 - 8}}{2}$$

$= -2 \pm \sqrt{2}$

29. $x > 0$, $y >$ –0.0061

$$\frac{\sqrt{x}\left(\frac{1}{x}\right) - \ln x\left(\frac{1}{2}x^{-\frac{1}{2}}\right)}{x} = 0$$

$\frac{1}{\sqrt{x}} - \frac{\ln x}{2\sqrt{x}} = 0$

$2 - \ln x = 0$

$\ln x = 2$

$x = e^2$

30. $x > 0$

$$\frac{2x^{\frac{3}{2}}\left(-\frac{1}{x}\right)-(2-\ln x)\left(3x^{\frac{1}{2}}\right)}{\left(2x^{\frac{3}{2}}\right)^2}=0$$

$$-2x^{1/2}-6x^{1/2}+3x^{1/2}\ln x=0$$

$$x^{1/2}(-2-6+3\ln x)=0$$

$$x^{1/2}=0 \text{ or } 3\ln x=8$$

$$\underset{\text{extraneous}}{x=0} \text{ or } \ln x=\frac{8}{3}\Rightarrow x=e^{8/3}$$

31. $x > 0$

$$x\left(\frac{1}{x}\right)+\ln x=0$$

$$1+\ln x=0$$

$$\ln x=-1$$

$$x=e^{-1}$$

32. $x < 1$

$$-\frac{1}{2}\left(-\frac{1}{2}\right)(1-x)^{-\frac{3}{2}}(-1)=0$$

$$\frac{-1}{4(1-x)^{3/2}}=0$$

no solution

33. $x \in \mathbb{R}$

$$x^4\cdot 3(x-1)^2+(x-1)^3\cdot 4x^3=0$$

$$x^3(x-1)^2(7x+4)=0$$

$$x=0 \text{ or } x=1 \text{ or } x=\frac{4}{7}$$

34. $x \in \mathbb{R}$

$$24x^2+4x^3=0$$

$$4x^2(6+x)=0$$

$$x=0 \text{ or } x=-6$$

35. $x \in \mathbb{R}$

$$15x^4-15x^2=0$$

$$15x^2(x^2-1)=0$$

$$x=0 \text{ or } x=\pm 1$$

36. $x \in \mathbb{R}$

$$60x^3-30x=0$$

$$30x(2x^2-1)=0$$

$$x=0 \text{ or } x=\pm\sqrt{\frac{1}{2}}=\pm\frac{\sqrt{2}}{2}$$

37. $x \neq 0$

$$1-\frac{1}{x^{2/3}}=0$$

$$x^{2/3}-1=0$$

$$x^{2/3}=1$$

$$x=1^{3/2}=1$$

38. $x \neq 0$

$$\frac{2}{3}x^{-\frac{5}{3}}=0$$

$$\frac{2}{3x^{5/3}}=0$$

no solution

39. $x < 5$

$$x\cdot\frac{1}{2}(5-x)^{-\frac{1}{2}}(-1)+(5-x)^{\frac{1}{2}}=0$$

$$\frac{-x}{\sqrt{5-x}}+\sqrt{5-x}=0$$

$$-x+2(5-x)=0$$

$$10-3x=0$$

$$x=\frac{10}{3}$$

40. $x \neq k\pi$

$$2-\csc^2 x=0$$

$$2-\frac{1}{\sin^2 x}=0$$

$$2\sin^2 x-1=0$$

$$\sin^2 x=\frac{1}{2}$$

$$\sin x=\pm\sqrt{\frac{1}{2}}=\pm\frac{\sqrt{2}}{2}$$

$$Q1: x=\frac{\pi}{4}+2k\pi \qquad Q2: x=\frac{3\pi}{4}+2k\pi$$

$$Q3: x=\frac{5\pi}{4}+2k\pi \qquad Q4: x=\frac{7\pi}{4}+2k\pi$$

41. $x \neq k\pi$

$$-2\csc x\left(-\csc x\cot x\right)=0$$
$$\frac{-2}{\sin x}\left(\frac{-1}{\sin x}\right)\left(\frac{\cos x}{\sin x}\right)=0$$
$$\frac{2\cos x}{\sin^3 x}=0$$
$$\cos x=0$$
$$x=\pm\frac{\pi}{2}+2k\pi$$

42. $x \in \mathbb{R}$

$$-2\sin x+2\sin x\cos x=0$$
$$2\sin x\left(-1+\cos x\right)=0$$
$$2\sin x=0 \text{ or } \cos x-1=0$$
$$x=k\pi \quad \text{or} \quad \cos x=1$$
$$x=2k\pi$$

$x=k\pi$ describes all

43. $x \in \mathbb{R}$

$$-2\cos x+2\cos 2x=0$$
$$-2\cos x+2(2\cos^2 x-1)=0$$
$$-2\cos x+4\cos^2 x-2=0$$
$$4\cos^2 x-2\cos x-2=0$$
$$2\cos^2 x-\cos x-1=0$$
$$(2\cos x+1)(\cos x-1)=0$$
$$\cos x=-\frac{1}{2} \quad \text{or} \quad \cos x=1$$
$$x=\pm\frac{2\pi}{3}+2k\pi \text{ or } x=2k\pi$$

44. $x \in \mathbb{R}$

$$\frac{2x}{1+x^2}=0$$
$$2x=0$$
$$x=0$$

45. $x \in \mathbb{R}$

$$\frac{\left(1+x^2\right)(2)-2x(2x)}{\left(1+x^2\right)^2}=0$$
$$2+2x^2-4x^2=0$$
$$2-2x^2=0$$
$$x^2=1$$
$$x=\pm 1$$

46. $x \neq \pm 1$

$$\frac{\left(1-x^2\right)(2x)-\left(1-x^2\right)(-2x)}{\left(1-x^2\right)^2}=0$$
$$2x-2x^3+2x-2x^3=0$$
$$-4x^3+4x=0$$
$$-4x\left(x^2-1\right)=0$$
$$x=0 \text{ or } \underset{\text{both extraneous}}{x=\pm 1}$$

47. $x \neq \pm 1$

$$\frac{\left(1-x^2\right)^2\cdot 4-4x\cdot 2\left(1-x^2\right)(-2x)}{\left[\left(1-x^2\right)^2\right]^2}=0$$
$$4\left(1-2x^2+x^4\right)+16x^2(1-x^2)=0$$
$$4-8x^2+4x^4+16x^2-16x^4=0$$
$$-12x^4+8x^2+4=0$$
$$3x^4-2x^2-1=0$$
$$\left(3x^2+1\right)\left(x^2-1\right)=0$$
$$3x^2+1=0 \quad \text{or } x^2-1=0$$

No solution $x=\pm 1$ (not in the domain)

No Solution

48. $x \neq 1$

$$\frac{(x-1)^2(1)-x(2)(x-1)}{\left[(x-1)^2\right]^2}=0$$
$$\left(x^2-2x+1\right)-\left(2x^2-2x\right)=0$$
$$x^2-2x+1-2x^2+2x=0$$
$$-x^2+1=0$$
$$\underset{\text{extraneous}}{x=1} \quad \text{or } x=-1$$

49. $x \in \mathbb{R}$

$$-\cos x\left(e^{\cos x}\right)-\sin x\left(e^{\cos x}\right)(-\sin x)=0$$
$$e^{\cos x}\left(-\cos x+\sin^2 x\right)=0$$
$$-\cos x+\left(1-\cos^2 x\right)=0$$
$$-\cos^2 x-\cos x+1=0$$
$$\cos^2+\cos x-1=0$$
$$\cos x=\frac{-1\pm\sqrt{5}}{2}$$
$$\cos x_1=\frac{-1-\sqrt{5}}{2}\approx -1.618$$
no $\mathbb{R}$ solution for x_1
$$\cos x=\frac{-1+\sqrt{5}}{2}\approx 0.6180$$
$$x\approx 0.9046+2k\pi$$

50. $x \in \mathbb{R}$

$$x^2(x-1)^2(7x-4)+x^3\cdot 2(x-1)(7x-4)+x^3(x-1)^2\cdot 7=0$$
$$x^2(x-1)\left[(x-1)(7x-4)+2x(7x-4)+7x(x-1)\right]=0$$
$$x^2(x-1)\left[\left(7x^2-11x+4\right)+\left(14x^2-8x\right)+\left(7x^2-7x\right)\right]=0$$
$$x^2(x-1)\left[28x^2-26x+4\right]=0$$
$$x=0 \text{ or } x=1 \text{ or } x=\frac{26\pm\sqrt{676-448}}{56}=\frac{26\pm\sqrt{228}}{56}$$

51. $x<5$

$$\frac{2(5-x)^{\frac{1}{2}}(-3)-(10-3x)\cdot 2\left(\frac{1}{2}\right)(5-x)^{-\frac{1}{2}}(-1)}{\left(2\sqrt{5-x}\right)^2}=0$$
$$(5-x)^{-1/2}\left[-6(5-x)+(10-3x)\right]=0$$
$$-30+6x+10-3x=0$$
$$3x-20=0$$
$$x=\frac{20}{3}$$
extraneous

No solution