

Section 5.5 Additional Problems Solutions to Assigned Problems

#10, 12, 16, 20, 28, 32, 38, 44, 52, 60 (All solutions may be checked by differentiation.)

$$10. \int \frac{1}{x^2} \cos^2\left(\frac{1}{x}\right) dx$$

$$\text{Let } u = -\frac{1}{x}, \text{ then } du = \frac{1}{x^2}.$$

$$\begin{aligned} \int \frac{1}{x^2} \cos^2\left(\frac{1}{x}\right) dx &= \int \cos^2(-u) du \\ &= \int \cos^2 u du \quad [\cos \text{ is even} \Rightarrow \cos(-u) = \cos u] \end{aligned}$$

Because $\cos^2 u$ is not a basic form, use the half-angle identity for cosine to transform the integrand into an expression that is a basic form.

$$\text{Half-angle identity: } \cos^2 u = \frac{1}{2}(1 + \cos 2u)$$

$$\begin{aligned} \int \cos^2 u du &= \int \frac{1}{2}(1 + \cos 2u) du \\ &= \frac{1}{2} \int 1 + \cos 2u du \\ &= \frac{1}{2} \left[\int 1 du + \int \cos 2u du \right] \\ &= \frac{1}{2} \left[u + \frac{1}{2} \sin 2u + C \right] \\ &= \frac{1}{2} \left[-\frac{1}{x} + \frac{1}{2} \sin 2\left(-\frac{1}{x}\right) + C \right] \end{aligned}$$

The last expression above is one representation of the solution but it can be simplified. Use the fact that sine is an odd function to write $\sin 2\left(-\frac{1}{x}\right) = -\sin\left(2\frac{1}{x}\right)$ and then use the double-angle identity to write $-\sin\left(2\frac{1}{x}\right) = -2\sin\left(\frac{1}{x}\right)\cos\left(\frac{1}{x}\right)$.

$$\int \frac{1}{x^2} \cos^2\left(\frac{1}{x}\right) dx = \frac{1}{2} \left[-\frac{1}{x} + \frac{1}{2} \sin\left(-\frac{2}{x}\right) + C \right] = -\left[\frac{1}{2x} + \sin\left(\frac{1}{x}\right)\cos\left(\frac{1}{x}\right) \right] + C$$

$$12. \int \frac{dx}{\sqrt{5x+8}}$$

a. Let $u = 5x + 8$, then $du = 5dx$.

$$\int \frac{dx}{\sqrt{5x+8}} = \frac{1}{5} \int \frac{5dx}{\sqrt{5x+8}} = \frac{1}{5} \int u^{-1/2} du = \frac{1}{5} \left(\frac{u^{1/2}}{1/2} \right) + C = \frac{2}{5} (\sqrt{5x+8}) + C$$

b. Let $u = \sqrt{5x+8}$, then $du = \frac{1}{2}(5x+8)^{-1/2} (5) = \frac{5}{2}(5x+8)^{-1/2}$

$$\int \frac{dx}{\sqrt{5x+8}} = \frac{2}{5} \int \frac{5}{2}(5x+8)^{-1/2} dx = \frac{2}{5} \int du = \frac{2}{5} u + C = \frac{2}{5} \sqrt{5x+8} + C$$

$$16. \int \frac{3dx}{(2-x)^2}$$

Let $u = 2 - x$, then $du = -dx$.

$$\int \frac{3dx}{(2-x)^2} = -3 \int \frac{-dx}{(2-x)^2} = -3 \int u^{-2} du = -3 \frac{u^{-1}}{-1} + C = \frac{3}{2-x} + C$$

$$20. \int \frac{(1+\sqrt{x})^3}{\sqrt{x}} dx$$

Let $u = 1 + \sqrt{x}$, then $du = \frac{1}{2} x^{-1/2} dx = \frac{dx}{2\sqrt{x}}$

$$\int \frac{(1+\sqrt{x})^3}{\sqrt{x}} dx = 2 \int u^3 du = 2 \frac{u^4}{4} + C = \frac{1}{2} (1+\sqrt{x})^4 + C$$

$$28. \int x^{1/3} \sin(x^{4/3} - 8) dx$$

Let $u = x^{4/3} - 8$, then $du = \frac{4}{3} x^{1/3} dx$.

$$\int x^{1/3} \sin(x^{4/3} - 8) dx = \frac{3}{4} \int \sin u du = -\frac{3}{4} \cos u + C = -\frac{3}{4} \cos(x^{4/3} - 8) + C$$

$$32. \int \frac{\sec z \tan z}{\sqrt{\sec z}} dz$$

Let $u = \sec z$, then $du = \sec z \tan z dz$.

$$\int \frac{\sec z \tan z}{\sqrt{\sec z}} dz = \int u^{-1/2} du = \frac{u^{1/2}}{1/2} + C = 2\sqrt{\sec z} + C$$

$$38. \int x^3 \sqrt{x^2 + 1} dx$$

Let $u = x^2 + 1$, then $du = 2x dx$.

$$\begin{aligned} \int x^3 \sqrt{x^2 + 1} dx &= \int x^2 \sqrt{x^2 + 1} x dx = \frac{1}{2} \int (u-1) u^{1/2} du = \frac{1}{2} \int u^{3/2} - u^{1/2} du = \frac{1}{2} \left(\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2} \right) + C \\ &= \frac{1}{2} \left(\frac{2u^{5/2}}{5} - \frac{2u^{3/2}}{3} \right) + C = \frac{3u^{5/2} - 5u^{3/2}}{15} + C = \frac{u^{3/2}(3u-5)}{15} + C \\ &= \frac{(x^2+1)^{3/2}(3x^2+3-5)}{15} + C = \frac{(x^2+1)^{3/2}(3x^2-2)}{15} + C \end{aligned}$$

$$44. \int \frac{\ln \sqrt{t}}{t} dt$$

Let $u = \sqrt{t}$, then $du = \frac{1}{2} t^{-1/2} dt = \frac{dt}{2\sqrt{t}}$

$$\int \frac{\ln \sqrt{t}}{t} dt = \int \frac{\ln \sqrt{t}}{\sqrt{t}} \frac{dt}{\sqrt{t}} = 2 \int \frac{\ln \sqrt{t}}{\sqrt{t}} \frac{dt}{2\sqrt{t}} = 2 \int \frac{\ln u}{u} du = 2 \frac{(\ln u)^2}{2} + C = (\ln \sqrt{t})^2 + C$$

$$52. \int \frac{\sqrt{\tan^{-1} x}}{1+x^2} dx$$

$$\text{Let } u = \tan^{-1} x, \text{ then } du = \frac{1}{1+x^2} dx$$

$$\int \frac{\sqrt{\tan^{-1} x}}{1+x^2} dx = \int \sqrt{u} du = \frac{u^{3/2}}{3/2} + C = \frac{2}{3} (\tan^{-1} x)^{3/2} + C$$

$$60. \frac{dy}{dx} = 4x(x^2 + 8)^{-1/3}, \quad y(0) = 0$$

$$y = \int 4x(x^2 + 8)^{-1/3} dx$$

$$\text{Let } u = x^2 + 8, \text{ then } du = 2x dx$$

$$y = \int 4x(x^2 + 8)^{-1/3} dx = 2 \int u^{-1/3} du = 2 \frac{u^{2/3}}{2/3} + C = 3(x^2 + 8)^{2/3} + C$$

$$64. \frac{d^2 y}{dx^2} = 4 \sec^2 2x \tan 2x, \quad y'(0) = 4, \quad y(0) = -1$$

$$\frac{dy}{dx} = \int 4 \sec^2 2x \tan 2x dx$$

$$\text{Let } u = \sec 2x, \text{ then } du = 2 \sec 2x \tan 2x dx$$

$$\int 4 \sec^2 2x \tan 2x dx = 2 \int u du = 2 \frac{u^2}{2} + C = (\sec^2 2x) + C$$

$$y'(0) = 4 \Rightarrow 4 = \sec^2(2 \cdot 0) + C \Rightarrow 4 = 1 + C \Rightarrow C = 3$$

$$\frac{dy}{dx} = \sec^2 2x + 3$$

$$y = \int \sec^2 2x + 3 dx = \frac{1}{2} \tan 2x + 3x + C$$

$$y(0) = -1 \Rightarrow C = -1, \text{ so } y = \frac{1}{2} \tan 2x + 3x - 1$$