

2.3 Exercises

VOCABULARY CHECK: Fill in the blanks.

- The equation $2 \sin \theta + 1 = 0$ has the solutions $\theta = \frac{7\pi}{6} + 2n\pi$ and $\theta = \frac{11\pi}{6} + 2n\pi$, which are called _____ solutions.
- The equation $2 \tan^2 x - 3 \tan x + 1 = 0$ is a trigonometric equation that is of _____ type.
- A solution to an equation that does not satisfy the original equation is called an _____ solution.

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at www.Eduspace.com.

In Exercises 1–6, verify that the x -values are solutions of the equation.

1. $2 \cos x - 1 = 0$

(a) $x = \frac{\pi}{3}$ (b) $x = \frac{5\pi}{3}$

2. $\sec x - 2 = 0$

(a) $x = \frac{\pi}{3}$ (b) $x = \frac{5\pi}{3}$

3. $3 \tan^2 2x - 1 = 0$

(a) $x = \frac{\pi}{12}$ (b) $x = \frac{5\pi}{12}$

4. $2 \cos^2 4x - 1 = 0$

(a) $x = \frac{\pi}{16}$ (b) $x = \frac{3\pi}{16}$

5. $2 \sin^2 x - \sin x - 1 = 0$

(a) $x = \frac{\pi}{2}$ (b) $x = \frac{7\pi}{6}$

6. $\csc^4 x - 4 \csc^2 x = 0$

(a) $x = \frac{\pi}{6}$ (b) $x = \frac{5\pi}{6}$

In Exercises 7–20, solve the equation.

7. $2 \cos x + 1 = 0$

8. $2 \sin x + 1 = 0$

9. $\sqrt{3} \csc x - 2 = 0$

10. $\tan x + \sqrt{3} = 0$

11. $3 \sec^2 x - 4 = 0$

12. $3 \cot^2 x - 1 = 0$

13. $\sin x(\sin x + 1) = 0$

14. $(3 \tan^2 x - 1)(\tan^2 x - 3) = 0$

15. $4 \cos^2 x - 1 = 0$

16. $\sin^2 x = 3 \cos^2 x$

17. $2 \sin^2 2x = 1$

18. $\tan^2 3x = 3$

19. $\tan 3x(\tan x - 1) = 0$

20. $\cos 2x(2 \cos x + 1) = 0$

In Exercises 21–34, find all solutions of the equation in the interval $[0, 2\pi)$.

21. $\cos^3 x = \cos x$

22. $\sec^2 x - 1 = 0$

23. $3 \tan^3 x = \tan x$

24. $2 \sin^2 x = 2 + \cos x$

25. $\sec^2 x - \sec x = 2$

26. $\sec x \csc x = 2 \csc x$

27. $2 \sin x + \csc x = 0$

28. $\sec x + \tan x = 1$

29. $2 \cos^2 x + \cos x - 1 = 0$

30. $2 \sin^2 x + 3 \sin x + 1 = 0$

31. $2 \sec^2 x + \tan^2 x - 3 = 0$

32. $\cos x + \sin x \tan x = 2$

33. $\csc x + \cot x = 1$

34. $\sin x - 2 = \cos x - 2$

In Exercises 35–40, solve the multiple-angle equation.

35. $\cos 2x = \frac{1}{2}$

36. $\sin 2x = -\frac{\sqrt{3}}{2}$

37. $\tan 3x = 1$

38. $\sec 4x = 2$

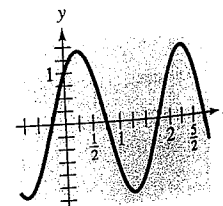
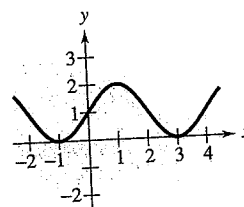
39. $\cos \frac{x}{2} = \frac{\sqrt{2}}{2}$

40. $\sin \frac{x}{2} = -\frac{\sqrt{3}}{2}$

In Exercises 41–44, find the x -intercepts of the graph.

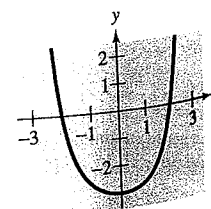
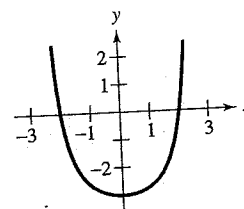
41. $y = \sin \frac{\pi x}{2} + 1$

42. $y = \sin \pi x + \cos \pi x$



43. $y = \tan^2\left(\frac{\pi x}{6}\right) - 3$

44. $y = \sec^4\left(\frac{\pi x}{8}\right) - 4$



In Exercises 45–54, use a graphing utility to approximate the solutions (to three decimal places) of the equation in the interval $[0, 2\pi)$.

45. $2 \sin x + \cos x = 0$
46. $4 \sin^3 x + 2 \sin^2 x - 2 \sin x - 1 = 0$
47. $\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = 4$
48. $\frac{\cos x \cot x}{1 - \sin x} = 3$
49. $x \tan x - 1 = 0$
50. $x \cos x - 1 = 0$
51. $\sec^2 x + 0.5 \tan x - 1 = 0$
52. $\csc^2 x + 0.5 \cot x - 5 = 0$
53. $2 \tan^2 x + 7 \tan x - 15 = 0$
54. $6 \sin^2 x - 7 \sin x + 2 = 0$

In Exercises 55–58, use the Quadratic Formula to solve the equation in the interval $[0, 2\pi)$. Then use a graphing utility to approximate the angle x .

55. $12 \sin^2 x - 13 \sin x + 3 = 0$
56. $3 \tan^2 x + 4 \tan x - 4 = 0$
57. $\tan^2 x + 3 \tan x + 1 = 0$
58. $4 \cos^2 x - 4 \cos x - 1 = 0$

In Exercises 59–62, use inverse functions where needed to find all solutions of the equation in the interval $[0, 2\pi)$.

59. $\tan^2 x - 6 \tan x + 5 = 0$
60. $\sec^2 x + \tan x - 3 = 0$
61. $2 \cos^2 x - 5 \cos x + 2 = 0$
62. $2 \sin^2 x - 7 \sin x + 3 = 0$

In Exercises 63 and 64, (a) use a graphing utility to graph the function and approximate the maximum and minimum points on the graph in the interval $[0, 2\pi)$, and (b) solve the trigonometric equation and demonstrate that its solutions are the x -coordinates of the maximum and minimum points of f . (Calculus is required to find the trigonometric equation.)

Function	Trigonometric Equation
63. $f(x) = \sin x + \cos x$	$\cos x - \sin x = 0$
64. $f(x) = 2 \sin x + \cos 2x$	$2 \cos x - 4 \sin x \cos x = 0$

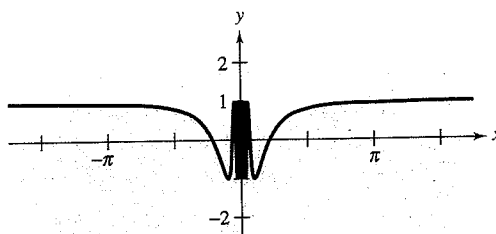
Fixed Point In Exercises 65 and 66, find the smallest positive fixed point of the function f . [A fixed point of a function f is a real number c such that $f(c) = c$.]

65. $f(x) = \tan \frac{\pi x}{4}$
66. $f(x) = \cos x$

67. Graphical Reasoning Consider the function given by

$$f(x) = \cos \frac{1}{x}$$

and its graph shown in the figure.



- (a) What is the domain of the function?
- (b) Identify any symmetry and any asymptotes of the graph.
- (c) Describe the behavior of the function as $x \rightarrow 0$.
- (d) How many solutions does the equation

$$\cos \frac{1}{x} = 0$$

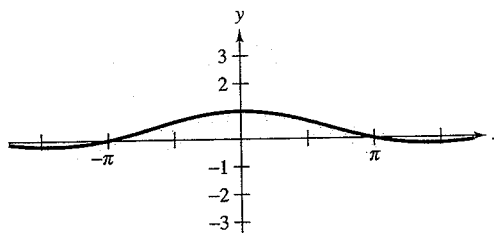
have in the interval $[-1, 1]$? Find the solutions.

- (e) Does the equation $\cos(1/x) = 0$ have a greatest solution? If so, approximate the solution. If not, explain why.

68. Graphical Reasoning Consider the function given by

$$f(x) = \frac{\sin x}{x}$$

and its graph shown in the figure.

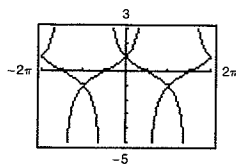


- (a) What is the domain of the function?
- (b) Identify any symmetry and any asymptotes of the graph.
- (c) Describe the behavior of the function as $x \rightarrow 0$.
- (d) How many solutions does the equation

$$\frac{\sin x}{x} = 0$$

have in the interval $[-8, 8]$? Find the solutions.

46. (a)



(b)

X	Y1	Y2
-3	-1.153	-.8676
-2	-4.588	-.518
-1	3.4022	2.934
0	ERROR	ERROR
1	-.2934	3.4022
2	2.934	-.518
3	-.8676	-1.153

Not an identity

(c) Answers will vary.

47-50. Answers will vary.

51. 1

52. 1 53. 2 54. 2

55. Answers will vary.

56. (a) Answers will vary.

(b)

θ	10°	20°	30°	40°	50°
s	28.36	13.74	8.66	5.96	4.20

θ	60°	70°	80°	90°
s	2.89	1.82	0.88	0

(c) Greatest: 10°; Least: 90° (d) Noon

57. False. An identity is an equation that is true for all real values of θ .

58. True. An identity is an equation that is true for all real values in the domain of the variable.

59. The equation is not an identity because $\sin \theta = \pm\sqrt{1 - \cos^2 \theta}$.

Possible answer: $\frac{7\pi}{4}$

60. The equation is not an identity because $\tan \theta = \pm\sqrt{\sec^2 \theta - 1}$.

Possible answer: $\frac{3\pi}{4}$

61. $-3 \pm \sqrt{21}$ 62. $\frac{-5 \pm \sqrt{53}}{2}$

63. $1 \pm \sqrt{5}$ 64. $\frac{1}{4}(1 \pm \sqrt{7})$

Section 2.3 (page 244)

Vocabulary Check (page 244)

1. general 2. quadratic 3. extraneous

1-6. Answers will vary. 7. $\frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi$

8. $\frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi$ 9. $\frac{\pi}{3} + 2n\pi, \frac{2\pi}{3} + 2n\pi$

10. $\frac{2\pi}{3} + n\pi$ 11. $\frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

12. $\frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi$ 13. $n\pi, \frac{3\pi}{2} + 2n\pi$

14. $\frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi, \frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi$

15. $\frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi$ 16. $\frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi$

17. $\frac{\pi}{8} + \frac{n\pi}{2}, \frac{3\pi}{8} + \frac{n\pi}{2}$ 18. $\frac{\pi}{9} + \frac{n\pi}{3}, \frac{2\pi}{9} + \frac{n\pi}{3}$

19. $\frac{n\pi}{3}, \frac{\pi}{4} + n\pi$ 20. $\frac{\pi}{4} + \frac{n\pi}{2}, \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi$

21. $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ 22. $0, \pi$ 23. $0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

24. $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}$ 25. $\frac{\pi}{3}, \frac{5\pi}{3}, \pi$ 26. $\frac{\pi}{3}, \frac{5\pi}{3}$

27. No solution 28. 0 29. $\pi, \frac{\pi}{3}, \frac{5\pi}{3}$

30. $\frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$ 31. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ 32. $\frac{\pi}{3}, \frac{5\pi}{3}$

33. $\frac{\pi}{2}$ 34. $\frac{\pi}{4}, \frac{5\pi}{4}$ 35. $\frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi$

36. $\frac{2\pi}{3} + n\pi, \frac{5\pi}{6} + n\pi$ 37. $\frac{\pi}{12} + \frac{n\pi}{3}$

38. $\frac{\pi}{12} + \frac{n\pi}{2}, \frac{5\pi}{12} + \frac{n\pi}{2}$ 39. $\frac{\pi}{2} + 4n\pi, \frac{7\pi}{2} + 4n\pi$

40. $\frac{8\pi}{3} + 4n\pi, \frac{10\pi}{3} + 4n\pi$ 41. $-1 + 4n$

42. $\frac{3}{4} + n$ 43. $-2 + 6n, 2 + 6n$

44. $-2 + 8n, 2 + 8n$ 45. 2.678, 5.820

46. 0.785, 2.356, 3.665, 3.927, 5.498, 5.760

47. 1.047, 5.236 48. 0.524, 2.618 49. 0.860, 3.426

50. 4.917 51. 0, 2.678, 3.142, 5.820

52. 0.515, 2.726, 3.657, 5.868

53. 0.983, 1.768, 4.124, 4.910

54. 0.524, 0.730, 2.412, 2.618

55. 0.3398, 0.8481, 2.2935, 2.8018

56. 0.5880, 2.0344, 3.7296, 5.1760

57. 1.9357, 2.7767, 5.0773, 5.9183 58. 1.7794, 4.5038

59. $\frac{\pi}{4}, \frac{5\pi}{4}, \arctan 5, \arctan 5 + \pi$

60. $\arctan(-2) + \pi, \arctan(-2) + 2\pi, \frac{\pi}{4}, \frac{5\pi}{4}$

61. $\frac{\pi}{3}, \frac{5\pi}{3}$ 62. $\frac{\pi}{6}, \frac{5\pi}{6}$

63. (a) (b) $\frac{\pi}{4} \approx 0.7854$

$\frac{5\pi}{4} \approx 3.9270$

Maximum: (0.7854, 1.4142)

Minimum: (3.9270, -1.4142)