u-Substitution and Integration by Parts Problems

Evaluate each integral by using substitution or integration by parts.

1. $\int \frac{16x}{\sqrt{8x^2+1}}dx$

$u = 8x^2+1$

$du = 16x\,dx$

$= \int u^{-1/2}\,du$

$= 2u^{1/2}+C$

$= 2\sqrt{8x^2+1}+C$

2. $\int x\sin\frac{x}{2}dx$

u	dv
x	$\sin\frac{x}{2}$
dx	$-2\cos\frac{x}{2}$

$= -2\cos\frac{x}{2} - \int -2\cos\frac{x}{2}dx$

$= -2\cos\frac{x}{2} + 4\sin\frac{x}{2}$

3. $\int 3\sqrt{\sin v}\cos v\,dv$

$u = \sin v$

$du = \cos v\,dv$

$\int 3u^{1/2}du$

$= 3\left(\frac{2}{3}\right)u^{3/2}+C$

$= 2(\sin v)^{3/2}+C$

4. $\int_0^1 \frac{16x}{8x^2+2}dx$

$u = 8x^2+2$

$du = 16x\,dx$

$x = 0 \Rightarrow u = 2$

$x = 1 \Rightarrow u = 10$

$\int_2^{10}\frac{1}{u}du$

$= \ln|u|\Big|_2^{10}$

$= \ln 10 - \ln 2 \approx 1.6$

5. $\int \theta\cos\pi\theta\,d\theta$

u	dv
θ	$\cos\pi\theta\,d\theta$
$d\theta$	$\frac{1}{\pi}\sin\pi\theta$

$\frac{1}{\pi}\theta\sin\pi\theta - \frac{1}{\pi}\int \sin\pi\theta\,d\theta$

$= \frac{1}{\pi}\theta\sin\pi\theta + \frac{1}{\pi^2}\cos\pi\theta + C$

6. $\int \frac{dx}{\sqrt{x}(\sqrt{x}+1)}$

$u = \sqrt{x}+1$

$du = \frac{1}{2}x^{-1/2}dx = \frac{1}{2\sqrt{x}}dx$

$2\int \frac{1}{u}du$

$= 2\ln|u|+C$

$= 2\ln|\sqrt{x}+1|+C$

7. $\int \sec^2\frac{t}{3}\,dt$

$u = \frac{t}{3}$

$du = \frac{1}{3}dt$

$3\int \sec^2 u\,du$

$3\tan u + C$

$3\tan\frac{t}{3}+C$

8. $\int \frac{1}{\theta^2}\sin\frac{1}{\theta}\,d\theta$

$u = \frac{1}{\theta}$

$du = \frac{-1}{\theta^2}d\theta$

$\int -\sin u\,du$

$\cos u + C$

$\cos\frac{1}{\theta}+C$

9. $\int t^2\cos t\,dt$

u	dv
t^2	$\cos t\,dt$
$2t\,dt$	$\sin t$

$= t^2\sin t - \int 2t\sin t\,dt$

u	dv
$2t$	$\sin t\,dt$
$2dt$	$-\cos t$

$= t^2\sin t - \left[-2t\cos t - \int -2\cos t\,dt\right]$

$= t^2 + 2t\cos t - 2\sin t + C$

10. $\int x\sec^2 x\,dx$

$x\tan x - \ln|\sec x| + C$

11. $\int_0^1 x\sqrt{1-x}\,dx$

$\frac{4}{15}$

12. $\int \sin^{-1}y\,dy$

$x\sin^{-1}x + \sqrt{1-x^2}+C$

13. $\int te^{-t}\,dt$

$-te^{-t} - e^{-t} + C$

14. $\int \ln t\,dt$

$t\ln t - t + C$

15. $\int \frac{1-2t^3}{t^3}dt$

$-\frac{1}{2t^2} - 2t + C$

16. $\int e^{\tan x}\sec^2 x\,dx$

$e^{\tan x}+C$

17. $\int z(\ln z)^2\,dz$

$\frac{z^2}{2}(\ln z)^2-\frac{z^2}{2}\ln x+\frac{z^2}{4}+C$

18. $\int_1^{e^{\pi/3}}\frac{dx}{x\cos(\ln x)}$

$\ln\left|2+\sqrt{3}\right|\approx 1.32$

19. $\int_0^{\pi/2}\theta^2\sin 2\theta\,d\theta$

$\frac{\pi^2}{8}-\frac{1}{2}$

20. $\int \sin^{-1}y\,dy$

$y\sin^{-1}y-\sqrt{1-y^2}+C$

21. $\int \left(x^2-5x\right)e^x\,dx$

$e^x\left(x^2-7x+7\right)$

22. $\int \frac{t^2+3t+4}{t-2}dt$

$\frac{t^2}{2}+5t-6\ln|t-2|+C$

23. $\int_0^1\frac{x^3+x^2-x-3}{x+2}dx$

$\frac{5}{6}-5\ln 3+5\ln 2$

24. $\int \frac{e^{\sqrt{t}}}{\sqrt{t}}dt$

$2e^{\sqrt{t}}+C$

Evaluate each integral by completing the square and using a substitution to reduce it to a basic form.

25. $\int_1^2\frac{8}{x^2-2x+2}dx$

$\int_1^2\frac{8}{(x-1)^2+1}dx$

$\int_0^1\frac{8}{u^2+1}=8\tan^{-1}u\Big|_0^1$

$8\left[\tan^{-1}1-\tan^{-1}0\right]$

$8\left(\frac{\pi}{4}\right)=2\pi$

26. $\int_2^4\frac{2}{x^2-6x+10}dx$

π

27. $\int \frac{dt}{\sqrt{-t^2+4t-3}}$

$\sin^{-1}(t+2)+C$

Evaluate each integral by using algebra, a trig identity, and substitution to reduce it to a basic form.

28. $\int \cos^2 d\theta$

$\int\frac{1+\cos 2\theta}{2}d\theta$

$\frac{\theta}{2}+\frac{1}{4}\sin 2\theta+C$

29. $\int_0^{\pi}\sin^2 x\,dx$

$\frac{\pi}{2}$

30. $\int_0^{\pi/2}\frac{\cos^3}{\sqrt{\sin x}}dx$

$\frac{8}{5}$