Calc 2 Review of Basic Integration

Function	**Derivative**		**Integral**
x^n	nx^{n-1}	$\int x^n,\ n \neq -1$	$\dfrac{x^{n+1}}{n+1}+C$
$\ln\lvert x\rvert$	$\dfrac{1}{x}$	$\int \dfrac{1}{x}$	$\ln\lvert x\rvert + C$
$\sin x$	$\cos x$	$\int \cos x$	$\sin x + C$
$\cos x$	$-\sin x$	$\int \sin x$	$-\cos x + C$
$\tan x$	$\sec^2 x$	$\int \sec^2 x$	$\tan x + C$
$\sec x$	$\sec x \tan x$	$\int \sec x \tan x$	$\sec x + C$
$\csc x$	$-\csc x \cot x$	$\int \csc x \cot x$	$-\csc x + C$
$\cot x$	$-\csc^2 x$	$\int \csc^2 x$	$-\cot x + C$
e^x	e^x	$\int e^x$	$e^x + C$
b^x	$\ln b \cdot b^x$	$\int b^x$	$\dfrac{1}{\ln b} b^x + C$
$\log_b x$	$\dfrac{1}{x \ln b}$		
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$	$\int \dfrac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x + C$
$\cos^{-1} x$	$\dfrac{-1}{\sqrt{1-x^2}}$	$\int \dfrac{-1}{\sqrt{1-x^2}}$	$\cos^{-1} x + C$
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$	$\int \dfrac{1}{1+x^2}$	$\tan^{-1} x + C$
k (constant)	0	$\int k$	$kx + C$
kf	kf'	$\int kf$	$k\int f$
$f \pm g$	$f' \pm g'$	$\int f \pm g$	$\int f \pm \int g$
f/g	$\dfrac{gf' - fg'}{g^2}$		
fg	$fg' + gf'$	$\int fg$	Integration by parts
$(f \circ g)(x)$	$f'(g(x))\,g'(x)$	$\int f'(g(x))\,g'(x)dx$	u-substitution: Let $g(x) = u$, then $= \int f'(u)\,du$

Antiderivatives

Definition: An **antiderivative** of the function f is any function F for which $F' = f$. An antiderivative of the function f is denoted $\int f(x)dx$.

Theorem: If F is an **antiderivative** of the function f on an interval I, then the most general antiderivative of f on I is $F(x) + C$ where C is any constant.

Properties of the antiderivative:

1. **Theorem 1**: If F and G are two antiderivatives of f on an interval $[a, b]$, then there is a constant c, such that $F(x) = G(x) + c$

2. **Theorem 2**: If f and g are two functions with antiderivatives $\int f(x)dx$ and $\int g(x)dx$, then the following hold:
   a) $\int cf(x)dx = c\int f(x)dx$ for any constant c.
   b) $\int [f(x) + g(x)]\, dx = \int f(x)dx + \int g(x)dx$.
   c) $\int [f(x) - g(x)]\, dx = \int f(x)dx - \int g(x)dx$.

Application

If the acceleration of an object at any time is known and the initial values of the objects position, $s(0)$, and its initial velocity, $v(0)$, are known, then the objects position at any time can be found. This is true since:

If $s(t)$ is given then,	If $a(t)$ is given then,
$v(t) = s'(t)$	$v(t) = at + v(0)$
$a(t) = v'(t)$	$s(t) = a\frac{t^2}{2} + v(0)t + s(0)$

Notation: We agree to write $F(b) - F(a)$ as $F(x)\big|_a^b$.

Terminology: In the definite integral $\int_a^b f(x)dx$ and in the indefinite integral $\int f(x)dx$, $f(x)$ is called the **integrand**.

Theorem: The First Fundamental Theorem of Calculus. Suppose f is a continuous function defined on an interval $[a,b]$ and also that F is an antiderivative of f (so $F' = f$), then $\int_a^b f(x)dx = F(b) - F(a)$.

Theorem: The Second Fundamental Theorem of Calculus. Let f be a continuous function defined on an open interval containing the interval $[a,b]$. Define $G(x) = \int_a^x f(t)dt$ for $a \le x \le b$. Then G is a differentiable function on $[a,b]$ and its derivative is f, that is $G'(x) = f(x)$.

Corollary: Suppose f is a continuous function defined on an interval $[a,b]$. Then f is the derivative of some function.

The Definite Integral

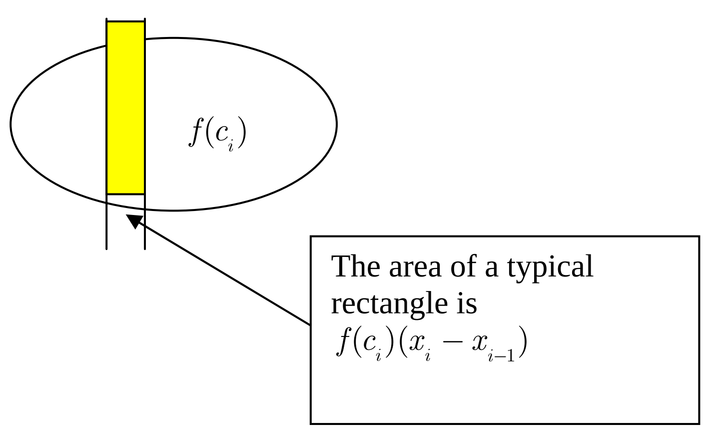
Definition: The definite integral of a function f over an interval $[a,b]$. If f is a function defined on an interval $[a,b]$ and the sums $\sum_{k=1}^{n} f(c_k)\Delta x_k = \sum_{k=1}^{n} f(c_k)(x_k - x_{k-1})$ approach a certain number as the length of all the intervals $[x_{k-1}, x_k]$ shrink towards 0 (regardless of the choice of c_k in each interval $[x_{k-1}, x_k]$), that number is called the **definite integral** of f over $[a,b]$.

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(c_k)\Delta x_k$$

The sum on the right-hand side is called a **Reimann sum**.

Interpretations of the Definite Integral

1) **Area of a plane region**: Area of $S = \int_a^b f(x)dx$ where $f(x)$ is the length of a cross section of S



x_{i-1} x_i

2) **Mass of a string**: Total Mass $= \int_a^b f(x)dx$, where $f(x)$ is the density of the string at the point x
3) **Distance traveled**: Total Distance $= \int_a^b f(t)dt$ where $f(t)$ is the velocity at time t
4) **The volume of a solid region**: Volume of $S = \int_a^b A(x)dx$, where $A(x)$ is the cross-sectional area at x
5) **Work**: If an object moves along a straight line by a force $f(x)$ that varies continuously, then the work, *W*, done in moving the object from $x = a$ to $x = b$ is $W = \int_a^b f(x)dx$