

Section 5.5 Additional Problems #46 Solution

46. Evaluate  $\int \frac{dx}{x\sqrt{x^4-1}}$ .

Let  $u = \sqrt{x^4-1}$ , so  $du = \frac{1}{2}(x^4-1)^{-1/2}(4x^3)dx = \frac{2x^3}{\sqrt{x^4-1}}dx$ .

Multiply both numerator and denominator of the original problem by  $2x^3$  to transform  $\int \frac{dx}{x\sqrt{x^4-1}}$  into

$$\frac{1}{2} \int \frac{2x^3}{x^4\sqrt{x^4-1}} dx$$

Use substitution, noting that  $u^2 = x^4 - 1$ , so  $x^4 = u^2 + 1$ . The expression for  $du$  is shown in red.

$$\begin{aligned} \frac{1}{2} \int \frac{2x^3}{x^4\sqrt{x^4-1}} dx &= \frac{1}{2} \int \frac{du}{u^2+1} = \frac{1}{2} \tan^{-1} u + C \\ &= \frac{1}{2} \tan^{-1} \sqrt{x^4-1} + C \end{aligned}$$