Section 5.5 Additional Problems Solutions to Assigned Problems

#10, 12, 16, 20, 28, 32, 38, 44, 52, 60 (All solutions may be checked by differentiation.)

10. $\int \frac{1}{x^2}\cos^2\left(\frac{1}{x}\right)dx$

Let $u = -\frac{1}{x}$, then $du = \frac{1}{x^2}$.

$$\int \frac{1}{x^2}\cos^2\left(\frac{1}{x}\right)dx = \int \cos^2(-u)\,du$$

$$= \int \cos^2 u\,du \qquad [\cos \textit{ is even} \Rightarrow \cos(-u) = \cos u]$$

Because $\cos^2 u$ is not a basic form, use the half-angle identity for cosine to transform the integrand into an expression that is a basic form.

Half-angle identity: $\cos^2 u = \frac{1}{2}(1+\cos 2u)$

$$\int \cos^2 u\,du = \int \frac{1}{2}(1+\cos 2u)du$$

$$= \frac{1}{2}\int 1+\cos 2u\,du$$

$$= \frac{1}{2}\left[\int 1\,du + \int \cos 2u\,du\right]$$

$$= \frac{1}{2}\left[u + \frac{1}{2}\sin 2u + C\right]$$

$$= \frac{1}{2}\left[-\frac{1}{x} + \frac{1}{2}\sin 2\left(-\frac{1}{x}\right) + C\right]$$

The last expression above is one representation of the solution but it can be simplified. Use the fact that sine is an odd function to write $\sin 2\left(-\frac{1}{x}\right) = -\sin\left(2\frac{1}{x}\right)$ and then use the double-angle identity to write $-\sin\left(2\frac{1}{x}\right) = -2\sin\left(\frac{1}{x}\right)\cos\left(\frac{1}{x}\right)$.

$$\int \frac{1}{x^2}\cos^2\left(\frac{1}{x}\right)dx = \frac{1}{2}\left[-\frac{1}{x} + \frac{1}{2}\sin\left(-\frac{2}{x}\right) + C\right] = -\left[\frac{1}{2x} + \sin\left(\frac{1}{x}\right)\cos\left(\frac{1}{x}\right)\right] + C$$

12. $\int \frac{dx}{\sqrt{5x+8}}$

a. Let $u = 5x+8$, then $du = 5dx$.

$$\int \frac{dx}{\sqrt{5x+8}} = \frac{1}{5}\int \frac{5dx}{\sqrt{5x+8}} = \frac{1}{5}\int u^{-1/2}du = \frac{1}{5}\left(\frac{u^{1/2}}{1/2}\right) + C = \frac{2}{5}\left(\sqrt{5x+8}\right) + C$$

b. Let $u = \sqrt{5x+8}$, then $du = \frac{1}{2}(5x+8)^{-1/2}(5) = \frac{5}{2}(5x+8)^{-1/2}$

$$\int \frac{dx}{\sqrt{5x+8}} = \frac{2}{5}\int \frac{5}{2}(5x+8)^{-1/2}dx = \frac{2}{5}\int du = \frac{2}{5}u + C = \frac{2}{5}\sqrt{5x+8} + C$$

16. $\int \frac{3dx}{(2-x)^2}$

Let $u = 2-x$, then $du = -dx$.

$$\int \frac{3dx}{(2-x)^2} = -3\int \frac{-dx}{(2-x)^2} = -3\int u^{-2}du = -3\frac{u^{-1}}{-1} + C = \frac{3}{2-x} + C$$

20. $\int \frac{\left(1+\sqrt{x}\right)^3}{\sqrt{x}}dx$

Let $u = 1+\sqrt{x}$, then $du = \frac{1}{2}x^{-1/2}dx = \frac{dx}{2\sqrt{x}}$

$$\int \frac{\left(1+\sqrt{x}\right)^3}{\sqrt{x}}dx = 2\int u^3du = 2\frac{u^4}{4} + C = \frac{1}{2}\left(1+\sqrt{x}\right)^4 + C$$

28. $\int x^{1/3}\sin\left(x^{4/3}-8\right)dx$

Let $u = x^{4/3}-8$, then $du = \frac{4}{3}x^{1/3}dx$.

$$\int x^{1/3}\sin\left(x^{4/3}-8\right)dx = \frac{3}{4}\int \sin u\, du = -\frac{3}{4}\cos u + C = -\frac{3}{4}\cos\left(x^{4/3}-8\right) + C$$

32. $\int \frac{\sec z \tan z}{\sqrt{\sec z}}dz$

Let $u = \sec z$, then $du = \sec z \tan z\, dz$.

$$\int \frac{\sec z \tan z}{\sqrt{\sec z}}dz = \int u^{-1/2}du = \frac{u^{1/2}}{1/2} + C = 2\sqrt{\sec z} + C$$

38. $\int x^3\sqrt{x^2+1}dx$

Let $u = x^2+1$, then $du = 2x\,dx$.

$$\int x^3\sqrt{x^2+1}dx = \int x^2\sqrt{x^2+1}\,xdx = \frac{1}{2}\int (u-1)u^{1/2}\,du = \frac{1}{2}\int u^{3/2}-u^{1/2}\,du = \frac{1}{2}\left(\frac{u^{5/2}}{5/2}-\frac{u^{3/2}}{3/2}\right)+C$$

$$= \frac{1}{2}\left(\frac{2u^{5/2}}{5}-\frac{2u^{3/2}}{3}\right)+C = \frac{3u^{5/2}-5u^{3/2}}{15}+C = \frac{u^{3/2}(3u-5)}{15}+C$$

$$= \frac{\left(x^2+1\right)^{3/2}\left(3x^2+3-5\right)}{15}+C = \frac{\left(x^2+1\right)^{3/2}\left(3x^2-2\right)}{15}+C$$

44. $\int \frac{\ln\sqrt{t}}{t}dt$

Let $u = \sqrt{t}$, then $du = \frac{1}{2}t^{-1/2}\,dt = \frac{dt}{2\sqrt{t}}$

$$\int \frac{\ln\sqrt{t}}{t}dt = \int \frac{\ln\sqrt{t}}{\sqrt{t}}\frac{dt}{\sqrt{t}} = 2\int \frac{\ln\sqrt{t}}{\sqrt{t}}\frac{dt}{2\sqrt{t}} = 2\int \frac{\ln u}{u}\,du = 2\frac{(\ln u)^2}{2} + C = \left(\ln\sqrt{t}\right)^2 + C$$

52. $\int \frac{\sqrt{\tan^{-1} x}}{1+x^2}\,dx$

Let $u = \tan^{-1} x$, then $du = \frac{1}{1+x^2}dx$

$$\int \frac{\sqrt{\tan^{-1} x}}{1+x^2}\,dx = \int \sqrt{u}\,du = \frac{u^{3/2}}{3/2} + C = \frac{2}{3}\left(\tan^{-1} x\right)^{3/2} + C$$

60. $\frac{dy}{dx} = 4x\left(x^2+8\right)^{-1/3}$, $y(0) = 0$

$$y = \int 4x\left(x^2+8\right)^{-1/3}\,dx$$

Let $u = x^2 + 8$, then $du = 2x\,dx$

$$y = \int 4x\left(x^2+8\right)^{-1/3}\,dx = 2\int u^{-1/3}\,du = 2\frac{u^{2/3}}{2/3} + C = 3\left(x^2+8\right)^{2/3} + C$$

64. $\frac{d^2y}{dx^2} = 4\sec^2 2x \tan 2x$, $y'(0) = 4$, $y(0) = -1$

$$\frac{dy}{dx} = \int 4\sec^2 2x \tan 2x\,dx$$

Let $u = \sec 2x$, then $du = 2\sec 2x \tan 2x\,dx$

$$\int 4\sec^2 2x \tan 2x\,dx = 2\int u\,du = 2\frac{u^2}{2} + C = \left(\sec^2 2x\right) + C$$

$$y'(0) = 4 \Rightarrow 4 = \sec^2(2\bullet 0) + C \Rightarrow 4 = 1 + C \Rightarrow C = 3$$

$$\frac{dy}{dx} = \sec^2 2x + 3$$

$$y = \int \sec^2 2x + 3\,dx = \frac{1}{2}\tan 2x + 3x + C$$

$y(0) = -1 \Rightarrow C = -1$, so $y = \frac{1}{2}\tan 2x + 3x - 1$