EXERCISES 5.5 ADDITIONAL PROBLEMS

Evaluating Integrals

Evaluate the indefinite integrals in Exercises 1–12 by using the given substitutions to reduce the integrals to standard form.

1. $\int \sin 3x\, dx, \quad u = 3x$

2. $\int x \sin(2x^2)\, dx, \quad u = 2x^2$

3. $\int \sec 2t \tan 2t\, dt, \quad u = 2t$

4. $\int \left(1 - \cos\frac{t}{2}\right)^2 \sin\frac{t}{2}\, dt, \quad u = 1 - \cos\frac{t}{2}$

5. $\int 28(7x - 2)^{-5}\, dx, \quad u = 7x - 2$

6. $\int x^3(x^4 - 1)^2\, dx, \quad u = x^4 - 1$

7. $\int \frac{9r^2\, dr}{\sqrt{1 - r^3}}, \quad u = 1 - r^3$

8. $\int 12(y^4 + 4y^2 + 1)^2(y^3 + 2y)\, dy, \quad u = y^4 + 4y^2 + 1$

9. $\int \sqrt{x} \sin^2(x^{3/2} - 1)\, dx, \quad u = x^{3/2} - 1$

10. $\int \frac{1}{x^2} \cos^2\left(\frac{1}{x}\right) dx, \quad u = -\frac{1}{x}$

11. $\int \csc^2 2\theta \cot 2\theta\, d\theta$

    a. Using $u = \cot 2\theta$ **b.** Using $u = \csc 2\theta$

12. $\int \frac{dx}{\sqrt{5x + 8}}$

    a. Using $u = 5x + 8$ **b.** Using $u = \sqrt{5x + 8}$

Evaluate the integrals in Exercises 13–54.

13. $\int \sqrt{3 - 2s}\, ds$

14. $\int (2x + 1)^3\, dx$

15. $\int \frac{1}{\sqrt{5s + 4}}\, ds$

16. $\int \frac{3\, dx}{(2 - x)^2}$

17. $\int \theta \sqrt[4]{1 - \theta^2}\, d\theta$

18. $\int \frac{4y\, dy}{\sqrt{2y^2 + 1}}$

19. $\int \frac{1}{\sqrt{x}(1 + \sqrt{x})^2}\, dx$

20. $\int \frac{(1 + \sqrt{x})^3}{\sqrt{x}}\, dx$

21. $\int \cos(3z + 4)\, dz$

22. $\int \tan^2 x \sec^2 x\, dx$

23. $\int \tan x\, dx$

24. $\int \tan^7 \frac{x}{2} \sec^2 \frac{x}{2}\, dx$

25. $\int r^2 \left(\frac{r^3}{18} - 1\right)^5 dr$

26. $\int r^4 \left(7 - \frac{r^5}{10}\right)^3 dr$

27. $\int x^{1/2} \sin(x^{3/2} + 1)\, dx$

28. $\int x^{1/3} \sin(x^{4/3} - 8)\, dx$

29. $\int \frac{\sin(2t + 1)}{\cos^2(2t + 1)}\, dt$

30. $\int \frac{6 \cos t}{(2 + \sin t)^3}\, dt$

31. $\int \frac{1}{\theta^2} \sin\frac{1}{\theta} \cos\frac{1}{\theta}\, d\theta$

32. $\int \frac{\sec z \tan z}{\sqrt{\sec z}}\, dz$

33. $\int \frac{1}{t^2} \cos\left(\frac{1}{t} - 1\right) dt$

34. $\int \frac{\cos\sqrt{\theta}}{\sqrt{\theta} \sin^2\sqrt{\theta}}\, d\theta$

35. $\int (s^3 + 2s^2 - 5s + 5)(3s^2 + 4s - 5)\, ds$

36. $\int t^3(1 + t^4)^3\, dt$

37. $\int \sqrt{\frac{x - 1}{x^5}}\, dx$

38. $\int x^3 \sqrt{x^2 + 1}\, dx$

39. $\int (\cos x)\, e^{\sin x}\, dx$

40. $\int (\sin 2\theta)\, e^{\sin^2\theta}\, d\theta$

41. $\int \frac{1}{\sqrt{x} e^{-\sqrt{x}}} \sec^2(e^{\sqrt{x}} + 1)\, dx$

42. $\int \frac{1}{x^2} e^{1/x} \sec(1 + e^{1/x}) \tan(1 + e^{1/x})\, dx$

43. $\int \frac{dx}{x \ln x}$

44. $\int \frac{\ln\sqrt{t}}{t}\, dt$

45. $\int \frac{dz}{1 + e^z}$

46. $\int \frac{dx}{x\sqrt{x^4 - 1}}$

47. $\int \frac{5}{9 + 4r^2}\, dr$

48. $\int \frac{1}{\sqrt{e^{2\theta} - 1}}\, d\theta$

49. $\int \frac{e^{\sin^{-1}x}\, dx}{\sqrt{1 - x^2}}$

50. $\int \frac{e^{\cos^{-1}x}\, dx}{\sqrt{1 - x^2}}$

51. $\int \frac{(\sin^{-1}x)^2\, dx}{\sqrt{1 - x^2}}$

52. $\int \frac{\sqrt{\tan^{-1}x}\, dx}{1 + x^2}$

53. $\int \frac{dy}{(\tan^{-1}y)(1 + y^2)}$

54. $\int \frac{dy}{(\sin^{-1}y)\sqrt{1 - y^2}}$

Initial Value Problems

Solve the initial value problems in Exercises 59–64.

59. $\frac{ds}{dt} = 12t(3t^2 - 1)^3, \quad s(1) = 3$

60. $\frac{dy}{dx} = 4x(x^2 + 8)^{-1/3}, \quad y(0) = 0$

61. $\frac{ds}{dt} = 8 \sin^2\left(t + \frac{\pi}{12}\right), \quad s(0) = 8$

62. $\frac{dr}{d\theta} = 3 \cos^2\left(\frac{\pi}{4} - \theta\right), \quad r(0) = \frac{\pi}{8}$

63. $\frac{d^2s}{dt^2} = -4 \sin\left(2t - \frac{\pi}{2}\right), \quad s'(0) = 100, \quad s(0) = 0$

64. $\frac{d^2y}{dx^2} = 4 \sec^2 2x \tan 2x, \quad y'(0) = 4, \quad y(0) = -1$

65. The velocity of a particle moving back and forth on a line is $v = ds/dt = 6 \sin 2t$ m/sec for all t. If $s = 0$ when $t = 0$, find the value of s when $t = \pi/2$ sec.

66. The acceleration of a particle moving back and forth on a line is $a = d^2s/dt^2 = \pi^2 \cos \pi t$ m/sec^2 for all t. If $s = 0$ and $v = 8$ m/sec when $t = 0$, find s when $t = 1$ sec.

Section 5.5, pp. 402–403 Additional Problems Answers

1. $-\frac{1}{3}\cos 3x + C$ 3. $\frac{1}{2}\sec 2t + C$ 5. $-(7x-2)^{-4} + C$
7. $-6(1-r^3)^{1/2} + C$
9. $\frac{1}{3}(x^{3/2}-1) - \frac{1}{6}\sin(2x^{3/2}-2) + C$
11. (a) $-\frac{1}{4}(\cot^2 2\theta) + C$ (b) $-\frac{1}{4}(\csc^2 2\theta) + C$
13. $-\frac{1}{3}(3-2s)^{3/2} + C$ 15. $\frac{2}{5}(5s+4)^{1/2} + C$
17. $-\frac{2}{5}(1-\theta^2)^{5/4} + C$ 19. $(-2/(1+\sqrt{x})) + C$
21. $\frac{1}{3}\sin(3z+4) + C$ 23. $\ln|\sec x| + C$
25. $\left(\frac{r^3}{18}-1\right)^6 + C$ 27. $-\frac{2}{3}\cos(x^{3/2}+1) + C$
29. $\frac{1}{2\cos(2t+1)} + C$ 31. $-\frac{\sin^2(1/\theta)}{2} + C$
33. $-\sin\left(\frac{1}{t}-1\right) + C$ 35. $\frac{(s^3+2s^2-5s+5)^2}{2} + C$
37. $\frac{2}{3}\left(1-\frac{1}{x}\right)^{3/2} + C$ 39. $e^{\sin x} + C$
41. $2\tan\left(e^{\sqrt{x}}+1\right) + C$ 43. $\ln|\ln x| + C$
45. $z - \ln(1+e^z) + C$ 47. $\frac{5}{6}\tan^{-1}\left(\frac{2r}{3}\right) + C$
49. $e^{\sin^{-1}x} + C$ 51. $\frac{1}{3}(\sin^{-1}x)^3 + C$ 53. $\ln|\tan^{-1}y| + C$
55. (a) $-\frac{6}{2+\tan^3 x} + C$ (b) $-\frac{6}{2+\tan^3 x} + C$
(c) $-\frac{6}{2+\tan^3 x} + C$
57. $\frac{1}{6}\sin\sqrt{3(2r-1)^2+6} + C$ 59. $s = \frac{1}{2}(3t^2-1)^4 - 5$
61. $s = 4t - 2\sin\left(2t+\frac{\pi}{6}\right) + 9$
63. $s = \sin\left(2t-\frac{\pi}{2}\right) + 100t + 1$ 65. 6 m 69. b) 399 Volts