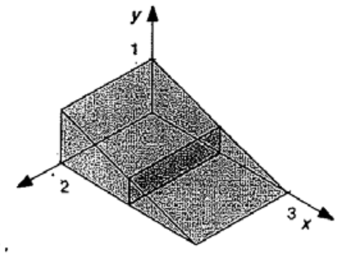
#1, 3, 4, 6, 10



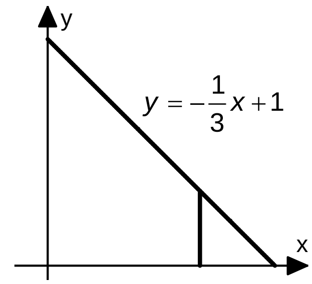
1. Cross sections are rectangles.

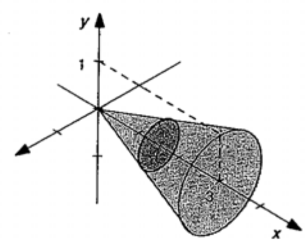
   Width = 2 and height is the *y*-value of the line in the *xy*-plane containing (0, 1) and (3, 0): $y = -\frac{1}{3}x + 1$

   $A = 2\left(-\frac{1}{3}x + 1\right)$

   $V_i = 2\left(-\frac{1}{3}x + 1\right)\Delta x$

   $$V = 2\int_0^3 -\frac{1}{3}x + 1\,dx$$
   $$= 2\left[-\frac{1}{6}x^2 + x\right]_0^3$$
   $$= 2\left[-\frac{9}{6} + 3\right]$$
   $= 3$ cubic units





3. Cross sections are circles:

   $Area = \pi r^2$, where *r* is the y-value on the line in the *xy*-plane containing (0, 1) and (3, 0): $y = -\frac{1}{3}x + 1$.

   $A = \pi\left(-\frac{1}{3}x + 1\right)^2$

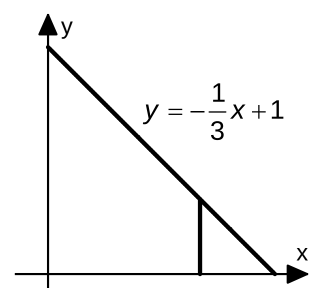
   $V_i = \pi\left(-\frac{1}{3}x + 1\right)^2 \Delta x$

   $$V = \pi\int_0^3 \left(-\frac{1}{3}x + 1\right)^2 dx$$

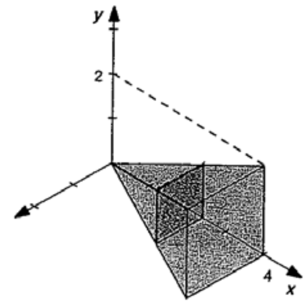
   Let $u = -\frac{1}{3}x + 1$, then $du = -\frac{1}{3}$.

   When $x = 0$, $u = 1$ and when $x = 3$, $u = 0$

   $$V = \pi\int_0^3 \left(-\frac{1}{3}x + 1\right)^2 dx$$
   $$= \pi(-3)\int_1^0 u^2 du$$
   $$= -3\pi\left(\frac{1}{3}u^2\right)_1^0$$
   $$= -3\pi\left(0 - \frac{1}{3}\right)$$
   $= \pi \approx 3.14$ cubic units



4. Cross sections are squares:



The length of a side is the *y*-value of the line in the *xy*-plane containing the points (0, 0) and (4, 2): $y = \frac{1}{2}x$.

$$Area = (\text{side})^2 = \left(\frac{1}{2}x\right)^2 = \frac{1}{4}x^2$$

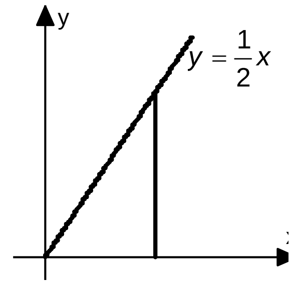
$$V_i = \frac{1}{4}x^2\Delta x$$

$$V = \int_0^4 \frac{1}{4}x^2 dx$$

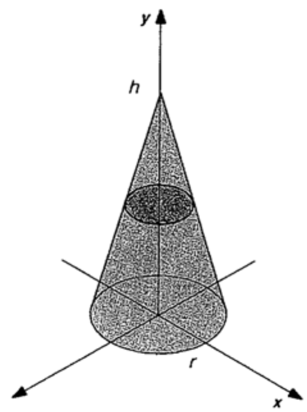
$$= \frac{1}{12}x^3\Big|_0^4$$

$$= \frac{1}{12}(64-0)$$

$$= \frac{64}{12} = \frac{16}{3} \quad \approx 5.33 \text{ cubic units}$$



6. Cross sections are circles:



The radius of each circle at level *y* is the *x*-value of the line in the *xy*-plane containing (0, *h*) and (*r*, 0): $y = -\frac{h}{r}x + h$. Because the slices are "*y* slices," solve the equation of the line for *x*:

$$x = -\frac{r}{h}(y-h).$$

$$Area = \pi(radius)^2$$

$$= \pi\left[-\frac{r}{h}(y-h)\right]^2$$

$$= \pi\frac{r^2}{h^2}(y-h)^2$$

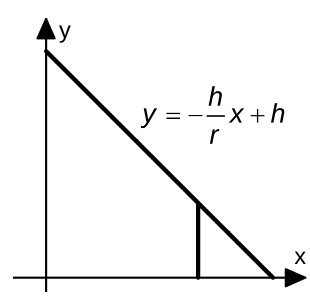
$$V_i = \pi\frac{r^2}{h^2}(y-h)^2 \ \Delta y$$

$$V = \frac{\pi r^2}{h^2}\int_0^h (y-h)^2 dy$$

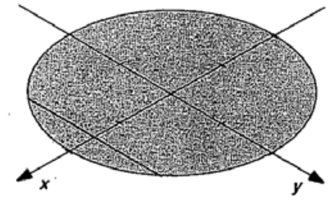
$$= \frac{\pi r^2}{h^2}\left(\frac{1}{3}\right)(y-h)^3\Big|_0^h$$

$$= \frac{\pi r^2}{3h^2}\left[0-(-h)^3\right]$$

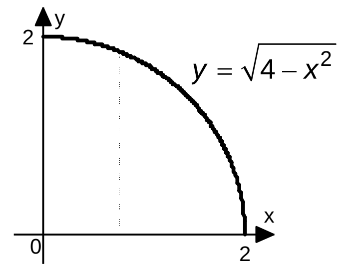
$$= \frac{1}{3}\pi r^2 h \text{ cubic units}$$



10. The base consists of the region in the *xy*-plane inside the circle $x^2 + y^2 = 4$. Cross sections perpendicular to the *x*-axis are (a) squares, (b) semicircles with diameter on the base, (c) equilateral triangles with one side on the base.



The length of a chord (a line seqment connecting two points of the circle) is *twice* the length of the *y*-value of the circle in the *xy*-plane: $y = \sqrt{4-x^2}$. The length of the chord is $2\sqrt{4-x^2}$.



(a) Cross sections are squares with side length $2\sqrt{4-x^2}$.

$$A = \left(2\sqrt{4-x^2}\right)^2 = 4\left(4-x^2\right)$$

$$V_i = 4\left(4-x^2\right)\Delta x$$

$$V = 4\int_{-2}^{2} 4 - x^2 dx = 8\int_{0}^{2} 4 - x^2 dx = 8\left(4x - \frac{1}{3}x^3\right)\Big|_0^2 = 8\left(8 - \frac{8}{3}\right)$$

$$= \frac{128}{3} \approx 42.67 \text{ cubic units}$$

(b) Cross sections are semicircles with diameter on the base. The diameter is $2\sqrt{4-x^2}$, so the radius is $\sqrt{4-x^2}$.

$$A = \frac{1}{2}\pi r^2 = \frac{\pi}{2}\left(\sqrt{4-x^2}\right)^2 = \frac{\pi}{2}\left(4-x^2\right)$$

$$V_i = \frac{\pi}{2}\left(4-x^2\right)\Delta x$$

$$V = \frac{\pi}{2}\int_{-2}^{2} 4 - x^2 dx = \pi\int_{0}^{2} 4 - x^2 dx = \pi\left(4x - \frac{1}{3}x^3\right)\Big|_0^2 = \pi\left(8 - \frac{8}{3}\right)$$

$$= \frac{16\pi}{3} \approx 16.76 \text{ cubic units}$$

(c) Cross sections are equilateral triangles with one side on the base:

The length of a side is $2\sqrt{4-x^2}$. The area of an equilateral triangle with side $2\sqrt{4-x^2}$ is

$$A = \frac{\sqrt{3}}{4}s^2 = \frac{\sqrt{3}}{4}\left[2\left(\sqrt{4-x^2}\right)\right]^2 = \frac{\sqrt{3}}{4}\left[4\left(4-x^2\right)^2\right] = \sqrt{3}\left(4-x^2\right)^2$$

$$V_i = \sqrt{3}\left(4-x^2\right)^2 \Delta x$$

$$V = \sqrt{3}\int_{-2}^{2} 16 - 8x^2 + x^4 \; dx = 2\sqrt{3}\int_{0}^{2} 16 - 8x^2 + x^4 \; dx$$

$$= 2\sqrt{3}\left(16x - \frac{8}{3}x^3 + \frac{1}{5}x^5\right)\Big|_0^2 = 2\sqrt{3}\left[32 - \frac{64}{3} + \frac{32}{5}\right]$$

$$= 2\sqrt{3}\left(\frac{256}{15}\right) = \frac{512}{15}\sqrt{3} \approx 59.12 \text{ cubic units}$$