Solution Section 7.3 #36

The air in a room with volume 180 m^3 contains 0.15% CO_2 initially. Fresher air with only 0.05% CO_2 flows into the room at a rate of $2\dfrac{m^3}{\text{min}}$ and the mixed air flows out at the same rate. Find the percentage of CO_2 in the room as a function of time. What happens in the long run?

Solution

Let *y*(*t*) be the amount of CO_2 in the room after *t* minutes. Then *y*(0) = 0.0015(180) = 0.27 m^3 of CO_2. The amount of air in the room is 180 m^3 at all times, so the percentage at time *t* (in minutes) is $p(t)=\dfrac{y}{180}\times 100\%$, and the change in the amount of CO_2 with respect to time is

$$\frac{dy}{dt}=(0.0005)\left(2\frac{m^3}{\text{min}}\right)-\frac{y(t)}{180}\left(2\frac{m^3}{\text{min}}\right)=0.001-\frac{y}{90}=\frac{9-100y}{9000}\frac{m^3}{\text{min}}$$

$$\int\frac{dy}{9-100y}=\int\frac{dt}{9000}$$

$$\ln|9-100y|=\frac{1}{9000}t+C$$

$$|9-100y|=Ce^{-t/90}$$

$$-100y=-9+Ce^{-t/90}$$

$$y=0.09+Ce^{-t/90}$$

Because *y*(0) = 0.27, $0.27=0.09+Ce^{-0/90}\quad\Rightarrow\quad C=0.27-0.09=0.18$

Therefore, $y=0.09+0.18e^{-t/90}$, and the percentage of CO_2 is given by

$$\begin{aligned}p(t)&=\frac{y}{180}\times 100\%\\&=\frac{0.09+0.18e^{-t/90}}{180}\times 100\%\\&=\left(0.0005+0.001e^{-t/90}\right)\times 100\%\\&=0.05+0.1e^{-t/90}\end{aligned}$$

In the long run, the percentage of CO_2 in the room will approach 0.05%.