Calculus Concepts and Contexts 3rd Edition by Stewart

Section 4.9 #54
Since raindrops grow as they fall, their surface area increases and therefore the resistance to their falling increases. A raindrop has an initial downward velocity of 10 m/s and its downward acceleration is

$$a = \begin{cases} 9 - 0.9t & 0 \le t \le 10 \\ 0 & t > 10 \end{cases}$$

If the raindrop is initially 500 m above the ground, how long does it take to fall?

Solution
Taking the upward direction to be positive, downward acceleration is given by

$$a = \begin{cases} -9 + 0.9t & 0 \le t \le 10 \\ 0 & t > 10 \end{cases}$$

Velocity is the antiderivative of acceleration, so

$$v = \begin{cases} -9t + 0.45t^2 + C_1 & 0 \le t \le 10 \\ C_2 & t > 10 \end{cases}$$

Because $v_0 = -10 = C_1$,

$$v = \begin{cases} -9t + 0.45t^2 - 10 & 0 \le t \le 10 \\ C_2 & t > 10 \end{cases}$$

Velocity is continuous, so the value of C_2 should be the same as v(10).

$$C_2 = v(10) = -(10) + 0.45(10)^2 = -55$$

and

$$v = \begin{cases} -9t + 0.45t^2 - 10 & 0 \le t \le 10 \\ -55 & t > 10 \end{cases}$$

Because position is the antiderivative of velocity and $s_0 = 500$,

$$s = \begin{cases} -4.5t^2 + 0.15t^3 - 10t + 500 & 0 \le t \le 10 \\ -55t + C_3 & t > 10 \end{cases}$$

After 10 seconds, the raindrop is at position $s(10) = -4.5(10)^2 + 0.15(10^3) - 10(10) + 500 = 100$ m.

(You don't need to find C_3 because you know how fast the raindrop is falling after 10 seconds (−55 m/s) and velocity is constant after 10 seconds.)

Find how long it takes the raindrop to fall the last 100 m when velocity is 55 m/s.

$$100 = 55t$$
$$t \approx 1.82$$

It takes 10 seconds to fall 400 m (to position 100m above the ground) and then about 1.82 seconds to fall the last 100 m, so it takes about 11.82 seconds for the raindrop to hit the ground.